Measurement and Macroscopicity

Overcoming Conceptual Imprecision in Quantum Measurement Theory

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0.1 Introduction

John S. Bell, of course, is most well known for the result now referred to simply as "Bell's theorem," which removed from consideration the class of so-called local hidden-variable theories which at the time of its publishing appeared to be the most natural class of theories among those which would render quantum mechanics a form of statistical mechanics. If, as this and further other results suggest, quantum theory is to serve as a truly *fundamental* theory, conceptual precision in its interpretation is not only desirable but paramount. John Bell was accordingly concerned about what he viewed as conceptual imprecision, from the physical point of view, within the standard approaches to the theory. He saw this as most acute in the case of their treatment of *measurement at the level of principle.* His concerns were strongly expressed in the article "Against Measurement." This item was published in 1990 in a volume of proceedings of the 1989 Erice meeting "Sixty-Two Years of Uncertainty," during which it was my pleasure to meet and eat with Bell, and to listen to him present this paper. He pointed out that this conceptual imprecision is reflected in the *terminology* of the foundations of quantum theory, a great deal of which he explicitly deemed worthy of banishment from discussions of principle, because it corresponds to a set of what he saw as vague and, in some instances, outright destructive concepts. His concern was thus not one regarding mathematics so much as regarding basic concepts used in contemporary quantum physics, which he viewed as failing the needs of natural philosophy and so of fundamental physics, despite their practical adequacy. Here, I consider John Bell's critique of standard quantum measurement theory and some alternative treatments wherein he saw greater conceptual precision, and make further suggestions as to how to improve conceptual precision, as he advocated.

That the source of difficulties is to be understood specifically as a problem of *imprecision of physical concepts* which stands in the way of the achievement of an exact fundamental mechanical theory is pointed out at the outset of "Against measurement": Bell wished to make it clear "at once that it is not mathematical precision, but physical" precision that caused him such great concern. As he saw it, one should have had by the time of its writing "an exact formulation of a serious part of mechanics," where by 'exact' he means "only that the theory should be fully formulated in mathematical terms, with nothing left to the discretion of the theoretical physicist," with non-relativistic 'particle' quantum mechanics and that of the electromagnetic field constituting a sufficiently "serious part" (Bell, 1991a). Bell also makes it immediately clear that he sees physics as part of the long tradition

of *natural philosophy*, and that his concerns about physical precision are, in effect, concerns regarding the *precision of concepts* of natural philosophy.

In the analysis offered in "Against Measurement," a key distinction made with regard to theoretical treatments compatible with experimental data obtained is that between those sufficiently precise to be accepted as fundamental physics and those good enough "for all practical purposes," for which he supplies the memorable acronym 'FAPP' (which, following his usage, continues to be employed in this sense now, two and one half decades later). Bell's exploration begins by pointing out that there is a lack of precision in the traditional, "proper treatments" one finds published in respectable and frequently consulted sources. He recalls and answers the often-asked rhetorical question of why one should bother making quantum mechanics more precise than it already is: "Why not look it up in a good book? But *which* good book? Usually the good unproblematic formulation is still in the head of the person in question... For the good books known to me are not much concerned with physical precision." His verdict on the various available treatments set firmly within quantum theory involving the standard, unmodified dynamical laws is that they are useful for practical predictions of the statistics to be found in experiments but fall far short of what physics ought to be *at the level of principle*: "The orthodox approaches, whether the authors think they have made derivations or assumptions, are just fine FAPP..." but ultimately fail to fully describe the physical world (Bell, 1991).

Bell makes provides a lengthy laundry list of standard quantum physical terms, reflection upon which shows that the lack of physical precision in the then current thinking—from which it should pointed out we have yet to significantly advance—is due to *conceptual* imprecision, and suggests that physics reject a considerable amount of its standard terminology.

"Here are some words which, however legitimate and necessary in application, have no place in a *formulation* with any pretension of physical precision: *system, apparatus, environment, microscopic, macroscopic, reversible, irreversible, observable, information, measurement.* The concepts of 'system', 'apparatus', 'environment', immediately imply an artificial division of the world, and an intention to neglect, or take only schematic account of, the interaction across the split. The notions of 'microscopic' and 'macroscopic' defy precise definition. So also do the notions of 'reversible' and 'irreversible'. Einstein said that it is theory which decides what is 'observable'. I think he was right — 'observation' is a complicated and theory-laden business. Then that notion should not appear in the *formulation* of fundamental theory." (Bell, 1991)

The ages old philosophical question of the relationship of observation to reality is relevant to the issues engaged in the article, but is itself not engaged

in the text in any detail, beyond a general rejection of subjectivism. It suffices here to note that in his writings generally, Bell sides with realism and cites one of its great physicist-champions. His primary concern instead is more specifically the relationship of *physical theory* to reality, and his position is that, at a minimum, physical theory should should explain to the physicist what can and cannot be measured, something given by answer to the question of how measurements are and can be made without themselves being considered *fundamental* to physical theory.

0.2 The negative influence of inappropriate terminology

As Bell sees it, "On this list of bad words from good books, the worst of all is 'measurement'." Again, he does not reject the term in general, particularly not its use in practice. He mentions, as an example, its use in the command "measure the mass and width of the Z boson" viewing such use as entirely acceptable; he objects, rather and most specifically, to "its use in the fundamental interpretive rules of quantum mechanics." When Bell considers as an example of the problems that arise through the use of 'measurement' in the foundations of quantum theory, he comments and answers rhetorically that, when reading of Dirac's "good book" *Quantum mechanics*, one gets the sense that "the theory is exclusively concerned about 'the results of measurement', and has nothing to say about anything else. What qualifies some physical systems to play the role of 'measurer'?" (Bell, 1991).

He suggests that the role of the notion of measurement be taken over by the more neutral term 'experiment': "Even in a low-brow practical account, I think it would be good to replace the word 'measurement', in the formulation, by the word 'experiment'. For the latter word is altogether less misleading," But, for him, this term also has limitations and, so, he continues,

"However, the idea that quantum mechanics, our most fundamental physical theory, is exclusively about the results of experiments would remain disappointing. ... To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise [of natural philosophy]. A serious formulation will not exclude the big world outside the laboratory." (Bell, 1991)

Thus, he finds use of the term 'experiment' tolerable in the formulation of quantum mechanics, but best avoided if possible.

'Measurement,' however, is "entirely inappropriate." Bell makes two specific charges against the term:

"The first charge against 'measurement', in the fundamental axioms of quantum mechanics, is that it anchors there the shifty split of the world into 'system' and 'apparatus'. A second charge is that the word comes loaded with meaning from everyday life, meaning which is entirely inappropriate in the quantum context ... In other contexts, physicists have been able to take words from everyday language and use them as technical terms with no great harm done ... Would that it were so with 'measurement'. But in fact the word has had such a damaging effect on the discussion, that I think it should now be banned altogether in quantum mechanics." (Bell, 1991)

Beyond the general difficulties of its use of this most problematic term, Bell sees the traditional, "orthodox" treatment of the measurement-like processes as reinforcing the imprecision of the enterprise of quantum natural philosophy in several ways. He also objects to any distinction between systems based on imprecise reference to *physical scale*, where the term 'macroscopic' is brought into play.

"The kinematics of the world, in this orthodox picture [with probabilities of obtaining outcomes], is given by a wavefunction (maybe more than one?) for the quantum part, and classical variables — variables that *have* values — for the classical part: $(\Psi(t, q, \ldots), X(t \ldots \ldots))$. The Xs are somehow macroscopic. This is not spelled out very explicitly. They dynamics is not very precisely formulated either." (Bell, 1991)

In this way, another to-be-banned term, 'macroscopic,' is indicated. Notably, Bell had expressed concern with the term 'macroscopic' in previous years as well, although he had previously seen a sharpening of the concept as still viable, particularly in that he commented that in regard to the 'EPR correlations' which violated his equality that he had "very little understanding of the position of... Bohr," which depended on restrictions on what is to be considered possible in measurements and made use of the term.

For example, in his 1981 article entitled "Bertlmann's socks and the nature of reality," he indicated as one possibility for progress in foundations of quantum theory that "it may be that Bohr's intuition was right—in that here is no reality below some 'classical' 'macroscopic' level. Then fundamental physical theory would remain fundamentally vague, until concepts like 'macroscopic' could be made sharper than they are today" (Bell, 1981a). In his comment to this paper made directly after its presentation Bell's contemporary and friend Abner Shimony who, along with John Clauser, Michael Horne, and Richard Holt provided a directly experimentally testable form of Bell's inequality (Clauser, Horne, Shimony and Holt, 1969), may have influenced Bell's thinking regarding measurement, in that he remarks there that

"perhaps I can help to focus on the source of the difficulty [in understanding Bohr's

answer to E.P.R.]. In any measuring process, Bohr insists upon a sharp distinction between object and subject. The apparatus is considered to be situated on the subject's side of this division. Hence it is characterized in terms of the concepts of everyday life (of which the concepts of classical physics are refinements). One may ask, however, whether it is possible to investigate the physical behavior of the apparatus. . . Bohr's answer is that. . . [it] is possible but then other apparatus will be employed in the investigation. The boundary between the object and the subject has shifted" (Bell, 1981a).

By the time of "Against measurement" Bell was referring to this division exactly as "the shifty split."

The focus of the critique in "Against measurement," is in fact most specifically on the conceptual imprecision involved in the treatment of *state evolution* during measurement that depends on the above (as Bell sees it) problematic system–apparatus division, which is typically made by having the apparatus, in one way or the other, qualify as 'macroscopic.' He notes that a range of different, often incompatible assumptions as to how a system can be considered to be macroscopic have been used in the standard approaches to quantum measurement (Jaeger, 2014). In "Against Measurement," Bell analyzes the traditional treatments, which invoke sudden changes of quantum state during experiments, making use of the following distinction "It will be convenient later to refer to. . . the *spontaneous* jump of a macroscopic system [S] into a definite configuration, as the [Landau–Lifschitz] LL jump. And the *forced* jump of a quantum system as a result of 'measurement' — *an external intervention* — as the Dirac jump." A "jump" placed in the same location as the latter appears in the formulation of von Neumann; the postulate of state collapse according to von Neumann is also noted in the text: "what vN actually *postulates* is that 'measurement' — an external intervention by [the rest of the world] R on S — causes the state $\sum_n c_n \phi_n$ to jump, with various probabilities into ϕ_1 *or* ϕ_2 *or*. . . From the 'or' here, replacing the 'and', as a result of external intervention, vN infers that the density matrix, averaged over the several possibilities, has no interference terms between states of the system which correspond to different measurement results." It is this invocation of measurement at the level of postulates that Bell finds extremely objectionable.

Von Neumann was clearly forced to postulate such a process, which he called "Process 1," pointing out its exceptional nature by noting that, on physical grounds, one would rather expect the more usual Process 2 to be the only one needed.

"[O]ne should expect that [Process 2] would suffice to describe the intervention caused by a measurement: Indeed, a physical intervention can be nothing else than

the temporary insertion of a certain energy coupling into the observed system, i.e., the introduction of an appropriate time dependency of $\hat{H}$. (Von Neumann, 1932)

Von Neumann had argued that the boundary between the measuring and measured systems should also be "arbitrary to a very large extent" (Von Neumann, 1955, p. 420) because whether the collapse happens to the measured system alone or to the joint system of measuring apparatus together with the measured system, the statistics of outcomes will be the same from the point of view of any physical system, such as a that of a human being, separate from them which becomes correlated with them in the same way a measurement is assumed to become during an experiment. Although this is true, it does not aid our understanding of what takes place during measurement, but instead leaves its details obscure. This is very clearly a case of a theory working, but working only "FAPP."

Bell then surveys other traditional treatments in other "good books," indicating the various instances of physical imprecision within them. In the case of the treatment of Landau and Lifschitz (LL), which "derive[s] the Dirac jump from the LL jump," he says "In the LL formulation... the theory is ambiguous in principle, about exactly when and exactly how the collapse occurs, about what is microscopic and what is macroscopic, what quantum and what classical." These, of course, are the most critical questions involved in a fuller understanding of the quantum mechanics of experimentation. For this reason, imprecision in relation to these obscures the problem itself, and makes it all the more difficult to solve.

Bell also considers the treatment of Kurt Gottfried, and offers natural suggestions for its missing details as part of an exploration of its more realistic character. Formally, he takes this as a treatment in which the density matrix ρ for the joint system of system $S' = S + A$, where A is the measurement apparatus system, is replaced by another density matrix $\hat{\rho}$, in which all non-diagonal elements are zero in the Hilbert space basis in which measured values and the apparatus 'pointer' variable values are to be perfectly correlated, something which is a prerequisite of an accurate measurement often postulated by realist (as well as operationalist) interpretations of quantum theory. To him, this appears to be the case because there is conceptual drift "away from the 'measurement' (... external intervention) orientation of orthodox quantum mechanics towards the idea that systems, such as S' above, have intrinsic properties — independently of and before observation. In particular, the readings of external apparatus are supposed to be really there before they are read," in that explication of measurement in which "KG derives, FAPP, the LL jump from assumptions at the shifted split R'/S', which

include a Dirac jump there," where $R' = R - A$. This is seen as having the advantage that some

"'macroscopic' 'physical attributes' *have* values at all times, with a dynamics that is related somehow to the butchering of ρ into $\hat{\rho}$ — which is seen as somehow not incompatible with the international Schrödinger equation of the system. Such a theory, assuming intrinsic properties, would not need external intervention, would not need the shifty split, but the retention of the vague word 'macroscopic' would reveal limited ambitions as regards precision." (Bell, 1991)

One might, he finally notes, avoid this term by introducing variables which have values even at small scales, as in the deBroglie–Bohm approach.

Bell had previously viewed this approach as important, in that it showed that "the subjectivity of the orthodox version, the necessary reference to the 'observer,' could be eliminated" (Bell, 1982). This indicates a potentially promising direction for increased conceptual precision, one which he sought to "publicize." In this earlier discussion, he drew three morals from the existence deBroglie–Bohm model: (1) "Always test your general reasoning against simple models," (2) "in physics the only observations we must consider on position observations, if only the positions of instrument pointers," and (3), one concerning terminology that was to be the main theme of "Against measurement." In the paper where these morals were drawn, "On the impossible pilot wave," Bell notes regarding (3) that "serious people" were likely "misled by the pernicious misuse of the word 'measurement'" which "strongly suggests the ascertaining of some preexisting property of some thing, any instrument involved playing a purely passive role. Quantum experiments are just not like that, as we learned especially from Bohr" (Bell, 1982).

However, as seen below, by the end of the 1980s, Bell found a different approach more promising, one that deviates from standard quantum mechanics at the level of law: Indeed, he had concluded already concluded by the time of his 1986 article "Are there quantum jumps?" that "If, with Schrödinger, we reject extra variables, then we must allow that his equation is not always right. . . it seems to me inescapable. . . a recent idea [of Ghirardi, Rimini, and Weber (Ghirardi, Rimini and Weber, 1985)], a specific form of spontaneous wavefunction collapse, is particularly simple and effective" (Bell, 1987).

0.3 Modified quantum dynamics

0.3.1 The desiderata and superpositions at large scales

In addition to their common use of notions of measurement and macroscopicity, standard analyses of quantum mechanical situations, as viewed from the perspective of data production, suffer from what has been called the "opportunistic employment of the superposition principle." That is, one is tempted to allow the superposition principle to operate whenever convenient and not operate whenever inconvenient, as opposed to understanding via specific basic quantities precisely when it may or may not be in force. This issue was taken up by Shimony in his article "Desiderata for a modified quantum dynamics," presented in a memorial session for Bell, wherein he *also* noted that "At a workshop at Amherst College in June *Bell* remarked that the stochastic modification of quantum dynamics is the most important new idea in the field of foundations of quantum mechanics during his professional lifetime" (Shimony, 1991).

It is clear in "Against Measurement" that Bell viewed the modification of the standard quantum dynamics and presence or absence of state superposition as important in that it is a move that provides an opportunity to correct at least some of the forms of imprecision noted above, by providing objectivity to the circumstances under which measurement-like events would or would not take place. It is, therefore, worth looking more closely at the context in which such theories can be developed. This is just want Shimony does just in "Desiderata...", by spelling out four assumptions "concerning the interpretation of the quantum mechanical formalism have the consequence of making the [measurement problem and the problem of Schrödinger's cat] so serious that it is difficult to envisage their solution without some modification of the formalism itself." These assumptions, variously sanctioned in "Against Measurement" (AM), are generally "strongly supported by physical and philosophical considerations, and therefore a high price would be paid by sacrificing one of them in order to hedge standard quantum mechanics against modifications." They are the following (Shimony, 1991).

(i) "The quantum state of a physical system is an objective characterization of it." (As Bell puts in AM, a "serious formulation will not exclude the big world outside the laboratory" and will not be concerned exclusively with "piddling laboratory operations.")

(ii) that connected with Bell's theorem: "The objective characterization of a physical system by its quantum state is complete, so that an ensemble of systems described by the same quantum state is homogeneous, without any differentiations stemming from differences in 'hidden variables.'"

(iii) "Quantum mechanics is the correct framework theory for all physical systems, macroscopic as well as microscopic, and hence it specifically applies to measuring apparatuses." (About which, however, it should be noted that "The main consideration in favor of [it being] the incompatibility proved by Bell (1987, pp.14-21 and 29-39) between quantum mechanics and local hidden variables theories, but Bell himself emphasizes that there is still an option of non-local hidden variables theories, which he does not regard as completely repugnant." Furthermore, this assumption has the implication that all variants of the Copenhagen interpretation are "ruled out.")

(iv) "At the conclusion of the physical stages of a measurement (and hence, specifically, before the mind of an observer is affected), a definite result occurs from among all those possible outcomes (potentialities) compatible with the initial state of the object." (Bell is skeptical even of having *biology* pertinent to measurement induction—hence his comment regarding collapse "Was the wavefunction of the world waiting to jump for thousands of years until a single-celled living creature appeared... or some better qualified system ... with a PhD?")

He then sets out a list of eight well supported desiderata for such a dynamics, the last pertaining critically to the proposal which Bell looked to, namely, that of GRW. It is

"The modified dynamics should be capable of accounting for the occurrence of definite outcomes of measurements performed with actual apparatus, not just with idealized models of apparatus. The Spontaneous Localization theory of [GRW '86] has been criticized for not satisfying this desideratum... Albert and Vaidman (Albert 1990, 156-8) note that the typical reaction of a measuring apparatus in practice is a burst of fluorescent radiation, or a pulse of voltage or current, and these are hard to subsume under the scheme of measurement of the Spontaneous Localization theory." (Shimony, 1991)

Shimony also notes difficulties in this approach—and others pertaining to Bell's concern about *irreversibility*—which Bell says also defies a precise conceptual basis: "a stochastic modification of quantum dynamics can hardly avoid introducing time-asymmetry. Consequently, it offers an explanation at the level of fundamental processes for the general phenomenon of irreversibility, instead of attempting to derive irreversibility from some aspect of complexity (which has the danger of confusing epistemological and ontological issues)."

Now, Bell was not the first to notice the imprecision of the traditional approach to quantum measurement. Notably, in 1970, Wigner—who had shown the limitations of von Neumann's arbitrariness of the location of the division involved in his own measurement schema by showing that if a cognitive systems is used as a measurement apparatus contradictions can appear (Wigner, 1963)—pointed this out very clearly in his critique of one highly

developed standard treatment: In what was perhaps the most sophisticated treatment within that approach, that of Danieri, Loingier and Prosperi, the three authors were said by him to be "using phrases such as 'macroscopic variables' and 'macroscopic objects' without giving a precise definition of these terms," (Freire, 2005) so that, for example, their premises could not be rigorously formulated.

"...[M]ost quantities which we believe to be able to measure, and surely all the very important quantities such as position, momentum, fail to commute with all the conserved quantities so that their measurement cannot be possible with a microscopic apparatus. This raises the suspicion that the macroscopic nature of the apparatus is necessary in principle and reminds us that our doubts concerning the validity of the superposition principle for the measurement process were connected with the macroscopic nature of the apparatus." (Wigner, 1971)

And this nature, now sometimes referred to by the term 'macroscopicity', is not rigorously characterized within that approach. Wigner noted that "the theory of the interaction of a quantum system with a classical (macroscopic) system has not been formulated so that the mathematical meaning of the arrows [indicating the change of joint-system state-vector upon measurement] is not clear." (Wigner, 1971, p. 7). In remarks on Prosperi's paper in a key meeting of measurement theorists at the outset of the 1970s, which continued along the lines of the DLP approach, Wigner noted specifically the inappropriateness of making use of "something as inadequately defined as is the macroscopic nature of something" in serious physical discussions (his remarks immediately follow the article of Prosperi (Prosperi, 1971)) (Freire, 2005).

In the years between Wigner's critique and Bell's later criticisms of traditional quantum measurement theory, Anthony Leggett had considered performing tests for the quantum effects in "macroscopic systems," preferably being large material systems, to better illuminate the question of a whether there is a clear role for macroscopicity in measurement. Leggett still wishes to find "evidence of a breakdown of the quantum mechanical scheme of the physical world [in] that which connects the world of atoms and electrons, for which it was originally developed, with the everyday world of our immediate experience," where quantum mechanically complementary properties appear compatible (Leggett, 2002). In particular, he wishes to find superposition and, so, interference effects that DLP had argued should not occur during measurements (cf. Home and Whittaker, 2002). For this purpose, Leggett has suggested studying superconducting devices (SQUIDs) and the Josephson effect, in which states of a current of electrons could, in principle, enter a superposition of states of clockwise and/or anti-clockwise

circulation (Leggett, 2000). Since the early work of Leggett, 'macroscopic' became increasing defined in terms large values of specific observable quantities, generalizing Bohr's original belief that heft and rigidity or those of others simply that a sufficiently large number of degrees of freedom are essential, and rather than being identified via criteria related to the resolution of naked eye, as the most direct understanding of the meaning of the term would suggest.

Leggett has proposed a measure he called the 'disconnectivity' D, a "semiquantitative" and "qualitatively defined" notion, claiming that "the quantum states important in the discussion of the [cat] paradox are characterized by a very high value of ... 'disconnectivity'; by contrast, the states necessary to explain so-called 'macroscopic quantum phenomena' in superfluids and superconductivity have only low disconnectivity, so that they are irrelevant to our question..." (Leggett, 1980). Rather, these center on the "most promising area to look [for high disconnectivity states is that of] phenomena where quantum tunneling plays an essential role" (Leggett, 1980). The GRW approach can be viewed as having participated in this trend as well.

0.3.2 Continuous spontaneous localization and beables

It was the tack of the work of GRW, shared by other workers such as Philip Pearle and conveniently called continuous spontaneous localization (CSL), that Bell saw as most clearly offering an alternative and "explicit model allowing a unified description of microscopic and macroscopic systems" with the starting point of "a modified quantum dynamics for the description of macroscopic objects..." in which systems of many components have wavefunctions that frequently spontaneously localize to small regions, claiming that with it "most features of the behavior of macroscopic objects are accounted for by quantum mechanics in a natural way, due to the irrelevant spreads of wave packets for macroscopic masses" (Ghirardi, Rimini and Weber, 1985). Regarding the behavior of their model, GRW remark that "If one assumes for simplicity that the localization frequencies λ_i of all microscopic (e.g., atomic) constituents of a macroscopic body are of the same magnitude..., the center of mass is affected by the same process with a frequency $\lambda_{\mathrm{macro}} = N\lambda_{\mathrm{micro}}$ ... where [the "macroscopic number"] N is of the order of Avogadro's number" (Ghirardi, Rimini and Weber, 1985).

In response to this work, Bell noted that

"In the GRW scheme this vagueness [regarding wavefunction collapse] is replaced by mathematical precision. ... departures of the Schrödinger equation show up very rarely and very weakly in few-particle systems. But in macroscopic systems, *as a*

consequence of the prescribed equations, pointers very rapidly point, and cats are very quickly killed *or* spared." (Bell, 1991).

Bell saw this aspect of the GRW approach in marked, positive contrast to examples of 'solutions' of the measurement problem involving infinite limits which had appeared, in particular, the 1972 model of Coleman and Hepp, where a solution for the dynamics of a model apparatus consisting of a semi-infinite array of spin-1/2 particles was given, which was viewed by some as a sort of solution to the measurement problem. He was critical of the Coleman–Hepp model, noting that, for it, "the rigorous reduction does not occur in physical time but only in an unattainable mathematical limit... the distinction is an important one" (Bell, 1975).

One difficulty subsequently encountered by the CSL approach is finding a set of parameters that allow it to describe what is observed. This difficulty is connected with what Bell called "beables," those quantities which could be understood realistically which could correspond with what is actually observed. Those beables associated with local space-time regions are "local beables." Bell viewed contemporary quantum mechanics textbooks as failing to focus on these quantities.

"What you may find there are the so-called 'local observables'. It is then implicit that the apparatus of the 'observation', or, better, of experimentation, and the experimental results are real and localized. We will have to do the best we can with these rather ill-defined local beables, while hoping always for a more serious reformulation of quantum mechanics where the local beables are explicit and mathematical and rather than implicit and vague" (Bell, 1990)

For their part, CSL theories have mainly followed what has been called the 'mass density ontology' (Allori, Goldstein, Tumulka and Zhangi, 2008), as evidenced, for example, by the use of the parameter N above, to which it would be proportional for systems built from a given sort of fundamental subsystem. For a state in a superposition of states with differing mass density for which there is an operator, the larger the difference of the mass density distribution of the states is, the more quickly a collapse will take place. Thus, the collapse rate for superpositions states of microscopic systems is small because the mass density differences are likewise small, and for superpositions of macroscopic states it is large because the mass density differences are likewise large.

However, there is a problem of persistent "tails" for any collapse process that completes in finite time: State functions correspondence to perfectly sharp values in position are not obtained in finite time in cases where sharp values are obtained and it is admitted that "[f]or a macrosystem, the pre-

cisely applied eigenstate–eigenvalue link does not work" (Pearle, 2009). At least one of the longest and most active advocates of CSL, Philip Pearle does not see his as precluding the prospect of success of the theory, despite Shimony's desideratum which regards tails specifically, namely,

"d. If a stochastic dynamical theory is used to account for the outcome of a measurement, it should not permit excessive indefiniteness of the outcome, where "excessive" is defined by considerations of sensory discrimination. This desideratum tolerates outcomes in which the apparatus variable does not have a sharp point value, but it does not tolerate 'tails' which are so broad that different parts of the range of the variable can be discriminated by the senses, even if very low probability amplitude is assigned to the tail. The reason for this intolerance is implicit in Assumption (iv)...If registration on the consciousness of the observer of the measurement outcome is more precise than the 'tail' indicates, then the physical part of the measurement process would not yield a satisfactory reduction of the initial superposition, and a part of the task of reducing the superposition would thereby be assigned to the mind. For this reason, I do not share the acquiescence to broad 'tails' that Pearle advocates (1990, pp. 203-4)..." (Shimony, 1991)

Pearle has argued more recently, as he had once in Bell's presence—at the same Amherst conference mentioned above—that "one should not express a new theory in an old theory's language," a comment "at which he beamed" (Pearle, 2009). In particular, Pearle argues that "a collapse theory is different from standard quantum theory and...therefore requires a new language, conceptual as well as terminological" (Pearle). Emphasis is put, for example, on "near possessed" rather than "possessed" values of physical quantities. In his explication of CSL, Pearle argues that

"CSL retains the classical notion that the physical state of a system corresponds to the state vector. Corresponding to a random field $w(\mathbf{x}, t)$ whose probability of occurrence is non-negligible, the dynamics always evolves a realizable state. Therefore, one is freed from requiring the (near) eigenstate-eigenvalue link criterion for the purpose of selecting the realizable states. I suggest that the eigenstate-eigenvalue link criterion be subsumed by a broader concept. It must be emphasized that this new conceptual structure is only applicable for a theory which hands you macroscopically sensible realizable states, not superpositions of such states. In the new language, corresponding to a quantum state, every variable possesses a distribution of values..." (Pearle, 2009)

The notions of this new 'language' is to be given meaning by considering ways in which it is to be used in context. The distribution here is not to be understood as a probability distribution, despite possessing all the defining properties of one, because in classical physics unlike in the case of the fundamental quantum context, argues Pearle, probabilities are understood as due to *ignorance*. One

"may give the name 'stuff' to a distribution's numerical magnitude at each value of the variable, as a generalization of Bell's quasi-biblical characterization, 'In the beginning, Schrödinger tried to interpret his wavefunction as giving somehow the density of the stuff of which the world was made.' One is encouraged to think of each variable's stuff distribution as something that is physically real. The notion allows retention of the classical idea that, for a physical state, every variable possesses an entity. What is different from classical ideas is that the entity is not a number." (Pearle, 2009)

Bell had used the term 'stuff' in the context of stochastic localization theory in AM:

"The GRW-type theories have nothing in their kinematics but wavefunctions. It gives the density (in a multi-dimensional configuration space!) to *stuff*. To account for the narrowness of that stuff in macroscopic dimensions, the linear Schrödinger equation has to be modified, in the GRW picture by a mathematically prescribed spontaneous collapse mechanism." (Bell, 1991)

On this view, *every variable* possesses such a distribution, so that "complementarity here means that variables whose operators do not commute do not possess joint distributions, but they do jointly possess distributions" (Pearle, 2009). As an example, one can consider a state that is the quantum superposition of two, one with state amplitude $\sqrt{1-\epsilon}$ and the other with amplitude $\sqrt{\epsilon}$. Under the above interpretation, the smaller "tail" state is to considered to represent "an unobservably small amount of stuff which allows describing the state vector by (qualified) possessed values assigned to macroscopic variables, consistent with the dominant state" (Pearle, 2009).

As noted above, a central problem for CSL is finding parameter ranges for which it would have experimental predictions deviating from that of standard quantum mechanics. Interference experiments are archetypical and would serve to differentiate the two, because CSL tends to destroy interference in that it naturally destroys one of the necessary pair distinct states in the case of massive systems, with an interference visibility that decreases with the increase in system mass. Thus, one might test the theory by considering, for example, two-slit experiments on each of a range of sorts of systems differing in their masses: photons, electrons, neutrons, atoms, and molecules. At one limit of this range, one has the photons, of course, are massless, on which CSL would have no additional effect. In the upper range of the experiments which have been performed, one finds the C-60 molecule, which has $N = 720$ nucleons. The value 720 is too small for a great impact, and is very much small that the Avagadro number often taken as a value one can say is clearly "macroscopic." Because collapse narrows wavepackets, it also leads to a momentum increase and hence to an energy increase,

requiring collapse rates that differ not only with particle number, but also particle mass (Pearle, 2009). Unfortunately, experiments capable of testing this hypothesis are not of the sort commonly performed and currently await testing.

0.4 Toward the removal of conceptual imprecision

In the quantum theory of measurement, experiments are typically understood schematically as follows. A system S is initially prepared in a quantum state T through a series of physical interactions, after which it is measured through interaction with an apparatus A which is required, in the process, to enter a state the value of the "pointer" property which itself becomes perfectly correlated with the value of the measured property E of S. A minimal requirement placed on a measurement is that a certain "calibration condition" be satisfied, namely, that if a property to be measured is a real one, then it should exhibit its value unambiguously and with certainty, cf. (Busch, Lahti and Mittelstaedt, , p. 28). For so-called "sharp observables," that is, properties represented by Hermitian operators, this calibration condition is equivalent to a probability reproducibility condition, namely, that a probability measure E_T for a property be "transcribed" onto that of the corresponding appartus pointer property. In addition, measurement is taken to include the reading of registered value in addition to the above process of registration of the measured property by apparatus A. The question of how this pointer "objectification" is achieved in view of the non-objectivity of the measured operator, is the first part of the so-called "objectification problem."

The second part of the objectivication problem is that of "value objectification." A pointer reading refers to the property value of the object system prior to measurement only if the measured observable was objective *before* the measurement. When the observable is non-objective, the question arises as to what happens to the system in the course of the measurement. In general, some state change is unavoidable. The attempts to minimize this irreducible 'disturbance' which then naturally lead to the concept of ideality of a measurement. Ideality requires another characteristic, namely, repeatability: A *repeatable* measurement will put the system in a state in which the pointer reading X refers to an objective value of the measured observable. This is taken to show that the existence of repeatable measurements is necessary for realistic interpretations of quantum mechanics (Busch and Jaeger, 2010). For such measurements, pointer objectification entails value objectification via a strong value correlation. Such an operational approach

to measurement, however, threatens to mask the objective, physical nature of measurements in themselves with which Bell was so concerned.

In his 1981 article "Quantum theory for cosmologists," Bell asked the following rhetorical questions about quantum measurements as understood within such a scheme.

"If [quantum] theory is to apply to anything but idealized laboratory operations, are we not obliged to admit that more or less 'measurement-like' processes are going on more or less all the time more or less everywhere? . . . The concept of measurement becomes so fuzzy that it is quite surprising to have it appearing in physical theory at the most fundamental level. . . [D]oes not any analysis of measurement require concepts more fundamental than measurement? And should not the fundamental theory be about these...?" (Bell, 1981b)

There is stark contrast between the everyday "classical" measurements and quantum measurements are considered. In classical physics, it *is* the case that situations that are more or less measurement like are going on all the time everywhere. The difficulty in the quantum case is that similar processes should be happening yet the experimental outcomes found by us are still consistent with the predictions made using the Schrödinger state evolution, although the superposition principle should not apply when such measurement-like processes are taking place, if human beings are treated just like other physical entities. If it is indeed the case that measurements, as distinct from subjective acts of observation, are nonetheless an integral part of physics and *not* artificially introduced, the special physical circumstances appearing in measurements must be circumscribed. In the search for physical clarity, we can remove anthropocentric elements of our conception of quantum measurement by finding the set of radically influential objects corresponding to this natural kind rather than a generic *apparatus* for measurements, and thus remove impediments toward progress in isolating the physical conditions underlying measurement as objective. Then, if human beings or other, larger sets of, for example, biological entities precipitate such physical conditions then the special role of these entities will have been objectively grounded and natural philosophy will have been advanced.

It may be helpful to consider the possibility that the set of related entities as being of a *natural kind*, because when successful, classes employed by science do correspond to natural kinds, such as has been done in the cases of the sorts of chemical element, subatomic particle, star, and galaxy. This does not require that these objects be treated differently from others, but only assists in our comprehending the implications of their set of common characteristics. In addition to having particular sets of natural properties in common with one another, these tokens should be subject to laws of nature

relevant to these properties. One can seek a set of conditions for being a member of the kind 'radically influential object' as a way of making progress toward an improved realist physics. From the formal point of view, such influencers are typically assumed to have these characteristics in common, they: i) induce non-unitary state-change, and ii) satisfy the conditions on the systems for providing a robust record of measurement outcomes. The first relates to the ability to disentangle the joint state of the influencer–target system and the latter corresponds to the production of Einsteinian elements of reality. However, Bell rightly pointed out that more fundamental properties should be present, of which these propertes are *consequences*. It is these configurations of fundamental properties that should play the key role in describing measurement via truly fundamental physical principles.

Some models of measurement, discussed further below, assume that measurements involve complex measuring systems with a large number of degrees of freedom prepared in metastable states. Some natural systems are known to us to measure—for example, our eyes when connected with our nervous system are such systems. Consider the human optical system taken to comprise all the material from the eyes to brain inclusively, while asking whether previously assumed characteristics are essential. We must look to our understanding of the behavior of macromolecules of the optical nervous system when light is incident upon it? Shimony has pointed out that the photoreceptor protein of the rod cells, known as rhodopsin, absorbs photon followed by a biochemical cascade that is then followed by an electrical pulse in the optic nerve. Rhodopsin has two components,

"retinal, which can absorb a photon, and opsin, which acts as an enzyme that effects the binding of about five hundred mediating molecules when it is triggered by the excited retinal. . . . [W]hat if the unitary dynamics of evolution of the photon and the retinal produces a superposition of the cis and the trans conformations? ...Would not such a superposition produce an indefiniteness of seeing or not seeing a visual flash, unless, of course, a reduction occurred further along the pathway from the optic nerve to the brain to the psyche?" (Shimony, 1991)

The distinct, alternative physical states corresponding to different conformations of a molecule which can superpose and then enter a specific state when in contact with the remainder of the nervous system with which it is in contact is central to the functioning of this light detection process taking place in a biochemical and electric realm. The presence of this larger subsystem beyond rhodopsin has an effect of "amplification." Some artificial systems can also mimic the behavior of such natural radical influencers, and so are exploitable by designers of experiments, e.g. avalanche photodiodes (APDs) plus electronics. These systems, for example, are rather complex

and involving many degrees of freedom and metastable initial states. Their effect appears to us to be completed in a way that, for example, a Stern–Gerlach magnet alone without a downstream beam-occupation detector, are not. Nonetheless, it has been argued by some, like Asher Peres (Peres, 1980), that amplication is not required for measurement to take place, an argument considered below in greater detail.

It is the resolution of questions regarding the requirements such as these that could provide the classification of the measurers as natural kind and would be a helpful element of any realist treatment of quantum state change, such as that called for by Bell. Although, like Bell, I am concerned about the notion of measurement being given an unusually prominent place in physics, I am much more concerned about notions associated with other terms that he criticizes in less detail in "Against measurement," such as 'observable' and 'observation,' that are clearly laden with not only the influence of theory, but more importantly with a directly *subjective*, that is, non-physical aspect. This should be kept in mind when asking the question of which characteristics of the above example are *necessary* to an objective understanding of measurement in contrast to those which may only be systematically present in the considerations of physicists only because they themselves are subjects.

Although at least one human knower is always *eventually* present who is witness in any successful experiment—or, in less contrived cases, simply observes ongoing natural events—that happens also to be large, this should not be allowed to beg the question of whether largeness is a necessary characteristic of a radical influencer in all data yielding situations. The matter of the necessity of amplification, which aids in the delivery of signals perceptible to the human senses, is similar subtle. Amplification has often been considered to lead to the objectivity of data produced because from the statistical point of view it makes it is very difficult to undo, providing many systems and observers to be affected by the resulting signal. Despite the presence of these characteristics in many or most familiar situations wherein one learns about the state of a part of the universe, one must ask whether there is an objective reason for requiring them in data yielding situations *in general.*

Another characteristic mentioned above that is often selected out for special status is *complexity.* Notably, it was indicated by DLP, who required in their treatment that interference "be absent by virtue of the complexity of the considered system," with complexity taken to refer to the number of degrees of freedom of the joint system being large (Danieri, Loinger and Prosperi, 1962). It is also strongly indicated as a requirement in the above mentioned work of Peres which "benefited from comments by J. S. Bell" entitled "Can we undo quantum measurements?" (Peres, 1980). Peres intro-

duces a simple measurement model and with it demonstrates that systems of
many degrees of freedom may effectively obey superselection rules because
it is impossible to measure any phase relationship between two quantum
states in the limit of an *infinite number* of degrees of freedom. He does
does so without requiring another common candidate requirement, namely,
a measurer prepared in metastable state. The demonstration is a sensible
on FAPP, but only FAPP, like many others. Its value is thus that it shows
that amplification and the presence of metastable states of the measuring
system are *not* necessary for measurement.

All the commonly assumed characteristics of measurers might be thought
by us to be natural simply because they reflect a subtle and unnoticed an-
thropocentrism, seeming natural to us only because we scientists are human.
Just because humans are comparative large physical systems and perform
measurements in the experimental context, does not mean that we must
have access to all measurement results: measurement and the experience
of a measurement are distinct, despite their going together in our human
experience. As Bell suggests, must ask ourselves what might be the purely
objective properties of measurement processes that could be solely respon-
sible for, or most significant to, the occurrence of every measurement.

We see above, for example, that having entered the field of a Stern–Gerlach
magnet alone is insufficient for the measurement of an appropriate parti-
cle's spin and that, following its action, there is a sufficient set of elements
present for a successful measurement to take place when detector plus elec-
tronics which capture the output beams which can later be viewed by an
experimenter. What is happening in this case? According to the Schrödinger
evolution, the effect of the magnetic field is only to entangle the particle's
spin with its direction; it is usually understood that it is the detector suite
that allows the spin to be identified after the above spin–path correlation
has occurred. The detector, unlike the magnetic field created by the mag-
net, is complex, as is the human nervous system. The one characteristic
that appears to survive our removal of unnecessary conditions is what ap-
pears to be something similar to physical *complexity*. It is helpful now to
recall, as Shimony noted, that a benefit of stochastic modification of quan-
tum dynamics was to offer an explanation ("for the general phenomenon of
irreversibility"), at the level of fundamental processes, something clearly in
harmony with Bell's call for a more fundamental notion than measurement
to account for the emergence of experimental data. We should, of course,
note Bell's warning that looking to "some aspect of complexity" has "the
danger of confusing epistemological and ontological issues." Proceeding cau-

tiously, then, we can ask in precisely what sense might the ontology of above situations be complex.

The commonly recognized measurement-like situations involve not only a significant number of degrees of freedom but can also be viewed as composed of a number of distinct parts. (Incidentally, the notion of natural kinds can also assist in distinguishing between internal and external degrees of freedom, via the notion of fundamental entities.) An important, well known example relevant to our considerations here is the Schrödinger cat experiment. In this thought experiment, Schrödinger considered an unstable atom A the decay of which would release a hammer H that, when released, would break a vial containing poison and allowing the cat C, which is taken as otherwise isolated from the rest of the universe, to be exposed to the poison (Schrödinger, 1935). Aside from the "absurdity" (as he calls it) of the appearance there of two equiprobable distinct states of the cat's 'health' at a particular moment in the corresponding overall state

$$|\Psi\rangle = 1\sqrt{2}(|\text{undecayed}\rangle_A|\text{unreleased}\rangle_H|\text{alive}\rangle_C + |\text{decayed}\rangle_A|\text{released}\rangle_H|\text{dead}\rangle_C),$$
$$(0.1)$$

of the joint system, this situation has been viewed (in the terminology to be replaced in a more precise conception of measurement), as linking the atom to the 'macroscopic' domain (cf. Jaeger, 2014). One can equally well, as Bell's position would relatively favor, view this as a situation involving complex physical circumstances. Clearly, the joint system of A+H+C is more complex in several respects than that of C taken alone. For example, it has more subsystems, more degrees of freedom, and involves an interaction of C with H. The same can be said for A+H relative to A+H+C, which although A+H already involves an interaction, involves additional interactions. But, C itself involves internal interactions. One can ask, for example, whether these overlooked internal interactions are significant in relation to the complexity of the situation or otherwise influence measurement, because no-one has ever observed the sort of alive–dead circumstances in a cat or similar being.

Finally, although the above example continues to be of conceptual importance, one is now in need of further, and more practical examples to make progress. On the side of practical, rather than thought experiments, let us consider as an example of what is now available to be explored, some experiments performed relatively recently by Gerlich et al. (Gerlich, Eibenberger, Tormandi et al., 2011). These experiments have been taken by their creators to "prove the quantum wave nature and delocalization of compounds composed of up to 430 atoms, with a maximal size of up to 60 Angstroms, masses up to $m = 6,910$ AMU." To us, they can be seen to involve some-

thing somewhat similar to the suprising predictions of quantum mechanics in the Schrödinger cat thought experiment. These quantum systems involve more than one thousand internal degrees of freedom.

An important point here is the factor in these experiments which differentiates the detector used from conventional detectors operating in the visible and near infrared, such as avalanche photodiodes and photomultiplier tubes of past quantum optics experiments, namely, that although those may be single-photon sensitive, they could not reliably determine the *number* of photons in a pulse of light like the photon-number-resolving detectors used in this experiment. Such deterimination is made possible as follows. The experiments used a calorimetry-based photon detector in which energy is deposited in an absorber whose thermometer was determined via an observed change in temperature. Tungsten transition-edge sensors were used and understood to operate in such a way that tungsten electrons act as both energy absorber and thermometer, and were prepared so as to keep the tungsten electrons on the edge of a superconducting-to-normal conduction transition; a dependence of resistance on temperature was set up so as to allow precise thermometry. The change of current in the voltage-biased detector was measured with a superconducting quantum-interference (SQUID) array and analyzed. The results obtained demonstrated quantum state superpositions of states of certain properties of large entities, suggesting that the number of degrees of freedom involved does not correspond to the sort of physical complexity required for induced behavior which would differ from that given by the standard quantum mechanical desciption.

The detailed study of such experiments, in particular, the array of instrumentation and its necessity for successfully providing data describing quantum phenomena, alongside the study of human sensory systems themselves, can be expected to yield a more refined understanding of meaurement-like processes within quantum theory and should provide novel insights allowing more precise vocabulary and concepts to be introduced to improve upon its basic principles, as John Bell recommended. Novel approaches to measurement in quantum optics, much as in the testing of the Bell–CSHS inequality itself in the past, can also aid us in transcendending the current limitations illustrated in the weakness which he identified in quantum physical terminology deployed in measurement situations. After such work, one will be in a better position to consider specific modifications of the fundamental laws of quantum physics or the quantum state description itself, both of which should help us progress in the direction that Bell suggested to us to go in order to advance natural philosophy.

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