Causality and quantum mechanics

D. J. Miller
Centre for Time, University of Sydney NSW 2006, Australia

Matt Farr
School of Historical and Philosophical Inquiry, The University of Queensland, Brisbane QLD 4072, Australia

I. INTRODUCTION

Is it possible for a cause to arise in a closed [1] quantum system? Is the structure of quantum mechanics compatible with retrocausality, i.e. causal influence of "earlier" events by "later" events as indexed by the direction of time on a clock in the laboratory in which the experiments are performed?

Details of the following will be provided in a paper on the arXiv [2].

II. THE SET-UP

As shown in Fig. 1, an ensemble of quantum systems are prepared in the maximally-mixed (MM) state I/n (Hilbert space dimensionality n), followed by measurement A (of a non-degenerate observable), operation C and measurement B (also non-degenerate). Firstly, let operation C cut off any causal link between A and B by randomizing the state after measurement A. Then the probabilities of outcomes of measurements A and B are $p^A[i] = p^B[j] = 1/n$ and $p^{AB}[i\&j] = 1/n^2$. We call this the neutral causal background (NCB). In the NCB, preparation as a cause has been eliminated because it is not to be changed from the maximum entropy state I/n and the whole ensemble is to be always included so pre- and post-selection of sub-ensembles, which are a potential source of (pseudo?)-causality, are not to be considered.

Now remove the above specification of C and let C be any operation. Operation C can be classified as

causal	$p^B[j] \neq 1/n$
retrocausal	$p^A[i] \neq 1/n$
bicausal	$p^A[i] \neq 1/n$ and $p^B[j] \neq 1/n$
acausal	$p^A[i] = p^B[j] = 1/n$
a correlator	$p^A[i] = p^B[j] = 1/n, p^{AB}[i\&j] \neq 1/n^2$

Note that being a "correlator" is weaker than being a cause. For example a cause provides a means of signalling between A and B but a correlator does not.

There is an asymmetry in this scenario because there is a sequence IACB in which measurement A is performed first (in the laboratory time frame) and B is performed second and IBCA in which measurement B is performed first and A is performed second. There are two consequences of this asymmetry. The first involves the structure of the theory. Measurements A and B and operation C can be specified schematically by a operators A, B and C. When the sequence is IACB, the key expression involves (schematically) terms of the form $\mathrm{Tr}(CAC^\dagger B) = \mathrm{Tr}(C^\dagger BCA)$ and for IBCA the key expression is $\mathrm{Tr}(CBC^\dagger A) = \mathrm{Tr}(C^\dagger ACB)$. We see that if $C = C^\dagger$ (e.g. for a projection measurement), the key expression is the same for both sequences but when $C \neq C^\dagger$ the order in the sequence matters. This structure is so fundamental that any change in it would constitute a new theory but we wish to investigate QM not a different theory. We shall see that this asymmetry in the formalism does not preclude retrocausality.

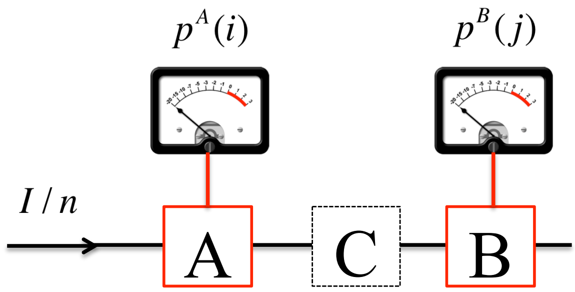

FIG. 1: An ensemble is prepared in a maximally mixed state and measurement A is performed with outcomes occurring with probabilities $p^A(i)$. The ensemble is then subjected to an operation C and then measurement B is performed with outcomes occurring with probabilities $p^B(j)$. If C can change $p^B(j)$, then C acts as a cause. If C can change $p^A(i)$, then C acts as a retrocause. Other possibilities are considered in the text.

The second aspect is that QM is usually derived from postulates which impose from the outset a completeness condition on the operators which specify a measurement. It turns out [2] that the completeness condition rules out the possibility of retrocausality so it would be preferable to leave the condition unspecified initially so that the possibilities for retrocausality can be investigated. Fortunately Barrett, Cresser, Jeffers and Pegg [3] have shown that QM can be derived beginning with a symmetric postulate which avoids (initially) the completeness condition.

III. RESULTS

Operation C is represented by a set of Kraus operators $\{C_k\}$. The completeness condition for them is $\sum_k C_k^\dagger C_k = I$.

A. Causality

Given that the experiment is performed in the laboratory in the sequence IACB, causality according to our definition involves the departure of $p^B[j]$ from its NCB value of $1/n$. When considering causality (but not, we shall see, when considering retrocausality which involves $p^A[i]$), we can impose the completeness condition on the $\{C_k\}$ while retaining the possibility of causation.

We show that C will be a cause if and only if

$$\sum_k C_k C_k^\dagger \neq I. \tag{1}$$

The crucial question is whether the operation C can be performed with $\sum_k C_k C_k^\dagger \neq I$ under the conditions imposed, namely only MM states as input and no pre- or post-selection. Operation C is closely related to a POVM. It is well known that any POVM can be carried out with an ancilla and a projective measurement but the ancilla must in a chosen state, not a MM state, and thus would represent an external source of causation and is contrary to our initial assumptions. For example with an ancilla in the state $|\psi\rangle$ (rather than the MM state as our condition of the NCB would require), we could simply switch our quantum system and the ancilla prior to measurement B. This would produce an effect according to our definition but the cause is obviously the state of the ancilla.

It is easy to implement the required C. One way [2] simply involves a polarising beam splitter (PBS) and a polarisation rotation on one mode through $\pi/2$ relative to the basis determined by the PBS. In fact all possible POVM's on single-photon polarisation states can be implemented by linear optics elements [4], ours is just a particular simple example. As Ahnert and Payne remark [4], the only ancilla required in their case (and our simplified example) is the vacuum. It does not seem appropriate to consider the vacuum as the cause of the effect here because the cause C can be changed by changing the orientation of the PBS while the vacuum remains the same. Therefore, although the vacuum could be considered an external system for our set-up, it cannot be considered the cause and so we consider it part of the NCB. Thus we conclude that the structure of QM allows for intrinsic causation.

B. Retrocausality

Next we examine $p^A[i]$ to see if QM can be intrinsically retrocausal. We find that operation C must violate the completeness condition $\sum_k C_k^\dagger C_k = I$. So the question becomes is it possible to devise an operation which does not satisfy the completeness condition which we will call an "incomplete operation". One can do so by post-selection, i.e. by discarding some events, but this is ruled out here because we require all events to be included. Normally the completeness condition is required to ensure probabilities sum to unity but that is not required when using the symmetric postulate of PBJ [3, 5]. However to obtain normal (non-retrocausal) QM from the symmetric postulate, PBJ require that $p^A[i]$ should be independent of subsequent measurements by adopting condition which is equivalent to a completeness condition on the C_k here, i.e. by, effectively, ruling out retrocausality which we do not want to do here. Of course, if the completeness condition does not apply one could signal superluminally using entangled systems but if retrocausality applies one would expect to be able to signal superluminally.

Therefore following the work of PBJ it seems that there is no fundamental reason why QM could not be extended so as to include retrocausality in certain circumstances. To achieve restrocausality it would be necessary to devise a quantum operation for which $\sum_k C_k^\dagger C_k \neq I$ without post-selection. In different language, one needs to implement a non-trace preserving completely positive map without post-selection. The operation would have to ensure that the quantum system was confined or steered to a subspace of the Hilbert space that initially applied to it. There does not seem to be a practical way of achieving that aim to date.

In summary, C is (i) acausal if $\sum_k C_k^\dagger C_k = I$ (conventional completeness condition) and $\sum_k C_k C_k^\dagger = I$, (ii) causal if $\sum_k C_k^\dagger C_k = I$ but $\sum_k C_k C_k^\dagger \neq I$, (iii) retrocausal if $\sum_k C_k^\dagger C_k \neq I$ but $\sum_k C_k C_k^\dagger = I$ and (iv) bicausal if $\sum_k C_k^\dagger C_k \neq I$ and $\sum_k C_k C_k^\dagger \neq I$. These criteria apply for both the IACB and IBCA sequences.

C. Correlation

Turning to the question of correlation, according to our classification scheme C is a correlator if (i) C is acausal, i.e. $\sum_k C_k^\dagger C_k = \sum_k C_k C_k^\dagger = I$ and (ii) $p^{AB}[i\&j]$ departs from its NCB value $1/n^2$. In other words, with C in operation, the marginal probabilities are not changed from the NCB but the joint probabilities are changed from the NCB values. When C is a correlator, there is no difference between the experiments IACB and IBCA. The correlation is *between* A and B but with no preferred direction from A to B versus B to A.

If one considers the correlation as being due to a quantum state, there is no reason to prefer the state propagating forwards in the laboratory time direction over backwards in laboratory time. These properties are reminiscent of the correlation in entangled quantum systems so the present results are related to entangled systems in the next section.

IV. COMPARISON WITH ENTANGLED SYSTEMS

For any bipartite state, there are various ways that an experiment on a single quantum system can be designed so that the joint and marginal probabilities between successive measurements on the single system mimic the probabilities of respective measurements on each member of the bipartite system.[6, 7]

The most general bipartite state, including entangled states, can be written

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \sum_{n,m=0,1} c_{nm} |l_n\rangle \otimes |r_m\rangle \tag{2}$$

where $\sum_n \sum_m |c_{nm}|^2 = 2$ (we have chosen this normalisation for convenience later) and $|l_n\rangle$ and $|r_m\rangle$ are orthonormal basis states propagating to the left and right shown in Fig. 2(a).

Measurements A and B on the bipartite system in Fig. 2(a) give the same results as the experiment IXCB, shown in Fig. 2(b), and experiment IYCA shown in Fig. 2(c). In Fig. 2(b), measurement B is as before but the first measurement is referred to as X because the Kraus operators for it will have to be chosen to be

$$X_i = A_i^T \tag{3}$$

where T denotes transpose in the basis used for the bipartite state. Since the A_i satisfy the completeness condition ($\sum_i A_i^\dagger A_i = I$)

$$\sum_i X_i X_i^\dagger = I \tag{4}$$

which, from Eq. (1), means that X cannot act as a cause.

With reference to Eq. (2), the single Kraus operator for C is

$$C_\Psi = \sum_{n,m=0,1} c_{nm} |r_m\rangle \otimes \langle l_n| \,. \tag{5}$$

In matrix form, the operation C is specified by

$$C_\Psi^\dagger C_\Psi = \begin{bmatrix} |c_{00}|^2 + |c_{10}|^2 & c_{00}^* c_{01} + c_{11} c_{10}^* \\ c_{00} c_{01}^* + c_{11}^* c_{10} & |c_{11}|^2 + |c_{01}|^2 \end{bmatrix} . \tag{6}$$

and $C_\Psi C_\Psi^\dagger = (C_\Psi^\dagger C_\Psi)^*$.

Consideration of the equivalent single qubit experiment allows one to understand the bipartite experiment in causal terms. Unless either $c_{01} = c_{10} = 0$ or $c_{00} = c_{11} = 0$ neither $C_\Psi^\dagger C_\Psi = I$ nor $C_\Psi C_\Psi^\dagger = I$ so, according to our definitions, operation C is bicausal. This is as expected: for these cases, the bipartite state $|\Psi\rangle$ is either a product state or a superposition of a product state and an entangled state and clearly a product state can produce deviations from the MM state at both A and B. Therefore the corresponding operation C in the single qubit equivalent case acts as both a cause and a retrocause. Although the preparation of a bipartite state can cause and/or correlate outcomes at measurements A and B, note again that it is not possible (to date, at least) to implement operation C with $C_\Psi C_\Psi^\dagger \neq I$ on a single quantum system without post-selection which then acts as the (extrinsic) source of the (merely apparent) causing of B and retrocausing of A (Fig. 2(b)) or vice versa (Fig. 2(c)).

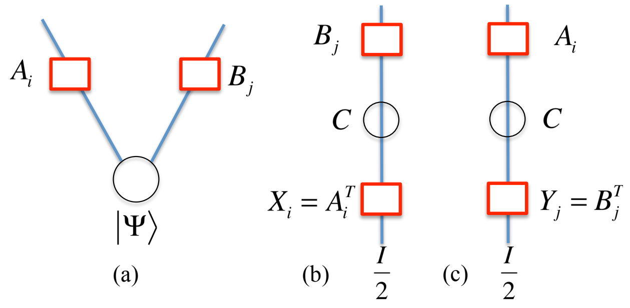

FIG. 2: The joint $p^{AB}[i\&j]$ and marginal probabilities $p^A[i]$ and $p^B[j]$ of experiments A and B in the three configurations are the same. (a) Bipartite state $|\Psi\rangle$ with measurement A (Kraus operators A_i) on one qubit and measurement B (Kraus operators B_j) on the other. (b) Single qubit prepared in the maximally-mixed state I. Measurement A now represented by Kraus operators A_i^T is performed first, then operation C which is related to the initial bipartite state $|\Psi\rangle$ (see text) and finally measurement B is the same as in (a). (c) Single qubit again prepared in the maximally-mixed state I. Now measurement B represented by Kraus operators B_j^T, operation C as in (b) and finally measurement A is the same as (a).

If either $c_{01} = c_{10} = 0$ or $c_{00} = c_{11} = 0$, $|\Psi\rangle$ is one of the maximally entangled, Bell states and $C_\Psi^\dagger C_\Psi = C_\Psi C_\Psi^\dagger = I$. For the Bell states, the reduced (unnormalised) state of each qubit is I and so the there is no deviation from the NCB and the source C of maximal entanglement is not a "cause". [10] On the other hand C is a correlator because even though the reduced states are I, there are stronger than classical correlations between the outcomes of A and B.

This means C is acausal and a correlator.

V. CONCLUSION

We have specified the conditions for causation in a closed quantum system. There is no reason in principle why retrocausation could not occur in a generalised form of quantum mechanics [3, 5] but there is no practical means of achieving that at present.

The concepts of causation and correlation should be more clearly distinguished. As we have defined it, a maximally entangled bipartite system is an example of pure correlation and not causation. Therefore analysis in terms normally used to deal with causation should not be applied to maximally entangled bipartite systems without re-consideration. In particular it seems appropriate to reconsider the application of Reichenbach's principle of common cause to that case. [8, 9] In particular, unlike causation in the above analysis, it is not appropriate to say that correlation operates in the laboratory direction of time.

[1] In this brief version we mean "closed" in the sense of the "closed" box of the Schrödinger cat experiment (decoherence is allowed). In the longer version [2], we show the same conclusions follow when "closed" includes the absence of decoherence.
[2] D. J. Miller and M. Farr, to be finalised.
[3] S. M. Barnett, J. D. Cresser, J. Jeffers and D. T. Pegg, New J. Phys. **16**, 043025 (2014).
[4] S. E. Ahnert and M. C. Payne, Phys. Rev. A **71**, 012330 (2005).
[5] D. T. Pegg, S. M. Barnett and J. Jeffers, Phys. Rev. A **66**, 022106 (2002).
[6] M. S. Leifer, Phys. Rev. A **74**, 042310 (2006).
[7] M. Laforest, R. Laflamme and J. Baugh, Phys. Rev. A **73**, 032323 (2006).
[8] E. G. Cavalcanti and R. Lal, J. Phys. A **47**, 424018 (2014).
[9] H. R. Brown and C. G. Timpson, arXiv:1501.03521 [quant-ph].
[10] Of course the source is a "cause" of events happening such as the outcome of experiments A and B registering. But it is not a "cause" in our sense in the Hilbert space of QM where the probabilities are determined because it does not result in a deviation from the MM state at either A or B.