

REVIEW OF:

TRAVIS NORSEN,
Foundations of Quantum Mechanics. An
Exploration of the Physical Meaning of
Quantum Theory.

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in Physics

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If you read one book (or a first book) on the subject indicated by its title, “foundations of quantum mechanics”, read this one. Unlike most other books about the mysteries of quantum mechanics (which are popularizations rather than serious physics texts, or which assume considerable technical background, or which are simply wrong-headed in their approach), Norsen’s book is very clearly written, is accessible to students with an undergraduate background in physics, and it covers all the important issues in that field.

Since this book contains a lot of “projects” namely problems to be solved by the reader, it can easily be used as a textbook for an advanced undergraduate course for physicists or for scientifically trained philosophers of science, and it came out from such a course given by the author.

Why might such a course be needed at all? That is, what, exactly, is the subject matter called the “foundations of quantum mechanics”, and why should anyone care? Perhaps it helps to compare to classical mechanics, which describes how bodies move under the action of forces. Now some forces, like the gravitational one, acting through the vacuum and at a distance, may seem quite mysterious (and that fact bothered Newton, who discovered them). One might wonder what those forces really are; similarly, if Newton’s laws are “laws of Nature”, one might wonder what a law of Nature is. These are questions that one might consider as belonging to the “foundations of classical mechanics”.

Working physicists might also dismiss these questions as “philosophical”. They have equations that tell them how bodies move, they can sometimes compute the solutions of those equations and compare those solutions with observations. They may say that this is all they are concerned with, and that they leave aside the ultimate nature of the terms entering their equations (the forces).

But in quantum mechanics the situation is radically different, even if most physicists do not realize it. Everybody knows how to use the quantum formalism in order to predict “results of measurements” and the (extraordinary) accuracy of those predictions is not in dispute.

But the textbook quantum formalism is remarkably silent about what the theory says outside of laboratories (where measurements are performed). Of course, physicists, in their vast majority at least, do believe that there is a reality “out there”, meaning outside of laboratories, but it is not clear at all what, if anything, their most fundamental theory, quantum mechanics, says about that reality.

If one asks a physicist what it means for an electron “out there” to have a given quantum state¹, the only orthodox answer he can give is that, if one brings that electron in a laboratory and one interacts with it in specific ways, namely if one “measures” some of its properties, one will obtain one among several possible results, each of them having a well-defined probability.

But if one takes that answer literally, then nothing is being said about the properties of the electron before it is introduced in the laboratory or about those of that overwhelming majority of electrons which are never introduced to any laboratory. And

¹By that we mean the wave function “times” (in the sense of tensors) some possible internal states, such as spin states.

if the poor electron has no property of its own, no position, no momentum, no energy, no angular momentum, etc. then what does it even mean to say that it exists “out there”?

That is why some physicists have fallen into an idealist sounding language (à la Bishop Berkeley: “to be is to be perceived”), that physics is fundamentally about observations, or information, or human perceptions. This sort of language is unique in the history of science. Consider classical mechanics again: Newton’s theory does not tell you that, if you look in the sky in a certain direction, you will see Mars or Jupiter. It tells you where these planets *are*. The statement about us seeing them if we look in the right direction may also be true, but requires some additional assumptions: that our telescopes work as advertised, that the sky is clear etc. These latter statements are not part of Newton’s theory, and, unlike quantum mechanics, that theory is not fundamentally about *observations*.

When people study physics, they usually want to learn about the “laws of Nature”. But if physics is all about observations, then how did these laws function throughout evolution in order to produce observers in the first place?

In fact, all this talk about observations confuses the ends and the means: we need observations to check our theories and not to fall into “metaphysics”, but the goal of our theories is to describe the world, not simply to predict “results of observations”.

This fundamental difference between the whole of pre-quantum physics and quantum mechanics, is sometimes called the measurement problem: why do measurements have such a central role in a fundamental physical theory? But it is better to call it, as Norsen and others do, *the ontology problem*: what does physics say about what exists “out there”?

So, Norsen devotes his first Chapter to a discussion of ontology in classical physics: in Newtonian mechanics, but also in Maxwellian electrodynamics. He also explain how and why “measurements” are *derived* notions in those theories.

After giving standard but fundamental examples of how quantum mechanics works, the particle in a box, the free Gaussian wave packet, the two slit experiment and the spin (Chapter 2), Norsen discusses how “measurements” work in quantum mechanics (Chapter 3), in a rather standard fashion, but showing that the collapse postulate of the quantum state cannot be brushed away, as illustrated by the famous Schrödinger’s cat thought experiment.

At this point, the reader may wonder if all the fuss about measurements is not simply a red herring: after all, the quantum state is a fundamentally probabilistic notion. If we measure a physical quantity of a system, then the word suggest that we *learn* something about the system being measured: why can’t measurements simply reveal pre-existing properties of the system? And, if that were so, why can’t the collapse of the quantum state simply be understood as the adjustment of probabilities after an observation? After all, if one throws a coin, then both faces have probabilities one-half before we look on which face it has fallen, but those probabilities jump to one or zero after we do look.

Isn’t quantum mechanics just like that? This is what Norsen calls, rightly, the *ignorance interpretation* of the quantum state: particles do have properties like position, momentum etc. before being measured, but we do not know them and the quantum

state simply encodes our partial knowledge and thus also our ignorance.

That brings us to the idea of “hidden variables”, discussed in section 3.4 of the book. Those variables would supplement the quantum description and would assign to individual particles properties that proper measurements would reveal. The quantum state could then simply be assigning a probability distribution over those variables. Even though the expression “hidden variables” is one of those physically incorrect words, so to speak, that people have been taught not to use in polite conversations about quantum mechanics, this view is probably in the back of the minds of the vast majority of physicists who are not bothered by the “measurement problem” in quantum mechanics.

And, if that view was tenable, there would indeed be no problem with quantum mechanics and, in particular, no ontology problem, since particles “out there” would have definite properties; the latter would simply be unknown to us before their measurement, but that is not very surprising.

Alas, once one tries to make that idea precise, one runs into a purely mathematical contradiction, because of the fundamental, but relatively unknown, no hidden variable theorems. The latter say that, if one assumes that a set of physical quantities have values that are revealed by measurements and if those quantities satisfy some relations that are implied by quantum mechanics (and verified empirically), then one can derive a contradiction (namely a statement of the form $0 = 1$).

So, the natural “ignorance interpretation” of quantum mechanics is untenable. Actually, a minor criticism that I would make about Norsen’s book is that the author does not delve enough into this fundamental issue of the no hidden variable theorems, although he does state clearly what they imply. Anyway, without an understanding of these theorems one cannot appreciate the depth of the conceptual problems posed by quantum mechanics.

But there is another side of the coin concerning those theorems: ever since von Neumann introduced their first version in his book on foundations of quantum mechanics [8], they have been often interpreted as vindicating the Copenhagen orthodoxy, namely that no description going beyond the quantum state can be introduced without running into a contradiction. But it all depends *which* hidden variables one introduces. As shown by Norsen, the pilot-wave theory (see below) introduces hidden variables *without* being contradicted by any of the no hidden variable theorems.

Coming back to the ontology problem, it only gets worse if one considers several particles: now, for a system of N particles in three dimensions, the wave function is a function on *configuration space*, namely a function of $3N$ variables, or an element of $L^2(\mathbb{R}^{3N})$.

How to make that function defined on configuration space correspond to anything in the real three dimensional world is, to say say to least, baffling. As Norsen puts it: “anyone who says that quantum mechanical wave functions should be taken seriously, as corresponding in some sense to physical reality (as opposed, for example, to our incomplete knowledge), should be asked to explain in concrete, mundane detail how that alleged correspondence works. They should tell us what sorts of things (particles?) or stuff (fields??) quantum mechanics is *about*, and clarify in precise mathematical detail the relationship between those things (and/or that stuff) and quantum mechanical wave functions.” (p. 124).

The idea of wave functions defined on configuration space leads naturally to the another problem emphasized by Norsen: *the locality problem*. Indeed, one can have a quantum state for two particles, labelled 1 and 2, of the form:

$$\Psi(x_1, x_2) = \frac{1}{\sqrt{2}}(\psi_1(x_1)\psi_2(x_2) \pm \psi_2(x_1)\psi_1(x_2)), \quad (1)$$

where ψ_1, ψ_2 are (normalized and orthogonal) single particle states and the choice of the sign $+$ or $-$ may depend on the example being considered. The two particles may be as far apart as one wishes.

Now, if one performs a measurement on particle 1 of a quantity represented by an operator A for which the states ψ_1, ψ_2 are eigenvectors, with eigenvalues λ_1, λ_2 , then, according to ordinary quantum mechanics, the state $\Psi(x_1, x_2)$ jumps to either $\psi_1(x_1)\psi_2(x_2)$ or $\psi_2(x_1)\psi_1(x_2)$, depending on whether one obtains λ_1 or λ_2 as a result of the measurement carried on particle 1.

But that means that the state of particle 2 changes also: before the measurement performed on particle 1 the value of the property associated to A was undetermined: it could be either λ_1 or λ_2 . After the measurement, it becomes with certainty λ_2 , if the collapsed state is $\psi_1(x_1)\psi_2(x_2)$, and with certainty λ_1 , if the collapsed state is $\psi_2(x_1)\psi_1(x_2)$.

So, if we know the result of the measurement on the first particle, we know with certainty the value of the property associated to A for the second particle.

But then, an obvious dilemma arises: was that property (for the second particle) already determined (but unknown to us) *before* the measurement performed on the first particle or was it created by that measurement?

If we give the second answer, we must admit, since the collapse is supposed to be instantaneous, that there are nonlocal causal effects or, to use a synonym, instantaneous actions at a distance in Nature.

This reasoning was basically the one of Einstein, and also of the famous Einstein, Podolsky and Rosen (EPR) 1935 paper [5]. But, since, for them, instantaneous actions at a distance were anathema, they thought that they had proven that quantum mechanics was incomplete, namely that the property associated to A for the second particle must pre-exist to the measurement performed on the first particle (and, by symmetry of reasoning, since one could do the measurement on the second particle without acting on the first particle, the property associated to A must pre-exist also for that first particle). Those pre-existing properties would again be instances of the so-called “hidden variables”.

But what we said above about the no hidden variable theorems, i.e. about the non-existence of properties pre-existing to their measurements, should make us suspicious about the first possibility (which is the only one that would allow us to deny the existence of instantaneous actions at a distance), namely that the property of the second particle pre-existed to the measurement performed on the first particle (and vice-versa).

And indeed, in 1964, almost 30 years after the EPR paper, John Bell proved that the mere assumption that such properties exist leads to a plain mathematical contradiction. So, the existence of nonlocal causal effects is the only logical conclusion of the

combination of EPR and Bell arguments.

This is extremely carefully discussed by Norsen, in his Chapters 4 and 8, including the various versions of the arguments of EPR and Bell and the experimental verifications of the quantum mechanical predictions used by Bell.

So, where are we? In order for quantum mechanics to say something about the “outside world”, we need a theory with an ontology or with “hidden variables”. However, the no hidden variable theorems tell us that one cannot introduce such an ontology in a naïve way, by assuming that measurements in general simply reveal pre-existing properties of particles. In fact, it would be nice if such a theory explained what measurements are and why they do not reveal pre-existing properties of quantum objects. Moreover, the EPR-Bell result also implies that such a theory must be non-local.

Surprisingly, there is a theory that exactly fits that bill, as old as or even older than ordinary quantum mechanics, and that Norsen explains in his Chapter 7; he calls it the pilot-wave theory, other people call it Bohmian mechanics or the de Broglie-Bohm theory since it was introduced, in an incomplete form, during the period 1924-1927 by de Broglie, then abandoned by him, but rediscovered and completed by Bohm in 1952 and further developed and promoted by John Bell [1], and also by Detlef Dürr, Sheldon Goldstein and Nino Zanghì [4].

In two words, the pilot-wave theory says: particles move! More precisely, their motion is guided by the wave function, through precise equations, a bit like classical particles are guided by the Hamiltonian of the system. In that theory, particles do have positions at all times, and thus also trajectories. Note that the particles positions are the hidden variables of the pilot-wave theory, but they are the only such variables, which is a crucial point, as we will see.

Norsen explains how this theory works, mostly through pedagogical examples, shows how it accounts for the famous double slit experiment and, most importantly, what “measurements” are in the pilot-wave theory. It turns out that all measurements of physical quantities other than positions are “contextual” in that theory, which means that their results are *not* determined by the properties of the particle alone (meaning its quantum state and the exact position of the particle) but by those properties *and* the precise way the measuring apparatus is set up. So that “measurements” do not measure any pre-existing property of particle, but are genuine interactions between a small object (the quantum particle) and a big one (the apparatus)². In other words, in the pilot-wave theory, there are no “hidden variables” associated to physical quantities (other than positions) that can be “measured”.

All this may sound paradoxical, since the pilot-wave theory is, after, all, completely

²If one analyzes along those lines what a “measurement of momentum” really is, one sees that there is no contradiction between the fact that particles have positions and velocities at all times and the Heisenberg uncertainty relations. Indeed the latter refer only to the statistics of “results of measurements” but a “measurement” of momentum is, in reality, an interaction with the particle, and one can show that the result of that interaction is *not* the actual momentum of the particle being measured, but some quantity whose statistical distribution satisfies the Heisenberg uncertainty relations. This is illustrated by Norsen with an example where each particle is initially at rest, yet the “measurement of momentum” yields a distribution in agreement with the quantum predictions, which are almost always non-zero.

deterministic. But that does not mean that the result of those specific interactions called measurements are determined by the initial conditions of the particle alone; they are determined once one specifies the initial conditions of the particle *and* of the apparatus with which it interacts. Norsen gives a novel example of this sort of contextuality: consider a particle whose wave function is a superposition of several energy eigenstates and let us “measure” its energy following some usual quantum mechanical procedure, which necessitates an interaction with the particle. While one might naïvely think that this measurement reveals “the” true energy of the particle, Norsen shows that the result, far from being a property of the particle alone, depends from the precise nature of that interaction. Therefore, there is simply no such thing as “the” pre-interaction energy of the particle.

So, here we have a theory that does have an ontology, that is not refuted by the no hidden variable theorems, and that explains what measurements are and why they do not reveal pre-existing properties of quantum objects. It is also non-local, as it should be. What else can we ask for?

Well, one could still look for other ways to solve the conceptual problems of quantum mechanics. And, despite Norsen’s obvious sympathy for the pilot-wave theory, he tends to bend himself backwards in order to give a fair hearing to two other such approaches: the spontaneous collapse theory (Chapter 9) and the many-worlds one (Chapter 10).

In the spontaneous collapse approach, one modifies the Schrödinger equations so that, at random times, the wave function collapses by being multiplied by a Gaussian function centered at a random point. Since these collapses are now part of the basic equations of the theory and not caused by “observations”, they are called spontaneous. As he does with other topics, Norsen presents probably the best short pedagogical introduction to the spontaneous collapse theory.

But since the wave function, even after a collapse, is a function defined on configuration space, we are still left with the problem of ontology. As Norsen explains, there are two standard “solutions” to that problem within the spontaneous collapse theory: one possibility is to assume that the wave function determines a density of mass in the real three dimensional space, which is an idea going back to Schrödinger (but which was abandoned because, without spontaneous collapses, in the “cat” example, one would get a mass density spread out between both cats³).

Another possibility is the so-called “flash ontology”, where the only reality are the space-time points around which collapses occur. A table or a measuring device or a human being is just a galaxy of such points.

Both ontologies are of course extremely bizarre: if one believes in the mass density ontology, one has to admit that atomic physics, with its picture of discrete objects, atoms, nuclei, electrons etc. was totally mistaken: all there is is some continuous density of stuff. For the flash ontology, the situation is even worse: since, in a finite universe, there were finitely many flashes since the Big Bang, most of the time, the universe is just empty!

Of course, one might respond that, with either of those ontologies, one is able to account for macroscopic observations and that this is all that one has to account for,

³That possibility is accepted in the “many-worlds” interpretation of quantum mechanics, see below.

since this is all that we have direct access to; nevertheless, one might hope that, to use Einstein's famous metaphor, God was not *so malicious* that he made us believe in the atomic theory of matter in a world where there are no atoms!

Moreover, the spontaneous collapse theory makes definite predictions that are at variance with the quantum mechanical ones. Thus, it can in principle be tested, but the parameters of the theory (the frequencies and the nature of the collapses) are chosen in such a way as to avoid being refuted, given the current technologies, which is not an appealing move, to put it mildly.

The other "theory" discussed by Norsen is the many-worlds one: just take the wave function evolving according to Schrödinger's equation and never collapse it! But then what happens to the famous dead-and-alive cat? Well, the world simply splits into two worlds: one where the cat is alive and a copy of myself sees the cat alive and one where the cat is dead and a copy of myself sees the cat dead. And it keeps on going like that, myriads of "worlds" proliferating by splitting into several copies each time a quantum event leads to a macroscopic superposed state.

Once more, Norsen does a very good job at explaining this view of things, including the two problems faced by this approach: again the one of ontology, since there is no obvious link between a wave function defined on configuration space and our familiar three dimensional experience. One can, just as for the spontaneous collapse theory, associate to the wave function a mass density in three dimensions, but that deprives the many-worlds approach from what is, in the eyes of its proponents, its main virtue: "ontological parsimony", meaning that the world is "only" the wave function. But, as explained in detail by Norsen throughout his book, this view is very difficult to make sense of.

A further problem with the many-worlds view is linked to Born's rule: suppose that "I" repeat many times the same experiment, with two possible outcomes, but having unequal probabilities, say $(\frac{1}{3}, \frac{2}{3})$. Then, after n repetitions of that experiment, there will be 2^n worlds, each one with a copy of me seeing a sequence of n results of the experiment. But, by the law of large numbers (since there are two results in each run of the experiment), in the vast majority of those worlds, the copies of me will see frequencies of each result occurring approximately half of the time and, therefore, they will stop believing in quantum mechanics after a while, since, according to quantum mechanics, they should see frequencies equal to $(\frac{1}{3}, \frac{2}{3})$.

This problem has been known to the many-worlders right from the beginning and many solutions have been proposed but in my (maybe not so modest) opinion, they are essentially verbal evasions, or, as Norsen puts it, have "a strong air of circularity": one attaches numbers to each of those worlds that fit the Born rule, but without linking those numbers to the statistics of the observations that our proliferating copies actually make.

Norsen calls the many-worlds theory a "work in progress" but that may be too charitable, given the lack of progress on this issue of the Born rule (not to speak of the question of ontology) since its invention in 1957 by Everett (who recognized the problem but evaded it).

If one compares the pilot-wave approach with the spontaneous collapse and the many-worlds ones, one is struck by how natural the first one is compared to the two

others: in the pilot-wave theory, all one has to accept is that there are particles and that particles means particles, namely localized objects moving in space. In the two other theories, one has to accept extravagant ontologies and either an undetectable violation of the quantum predictions (for the spontaneous collapse theory) or an inability to make sense of Born's rule (for the many-worlds approach).

Finally, Norsen must be credited with writing a very clear dissection of the "Copenhagen interpretation" (Chapter 6). He goes through the writings of the "founding fathers", Bohr and Heisenberg, noticing similarities and differences. Roughly speaking, in Bohr, there is a vaguely "Kantian" idea that we are forced, because of the nature of our minds, to use classical concepts, although we know that they are inadequate in the microscopic domain. Hence, the idea of complementarity: for example, we can use a particle language or a wave language, but not both at the same time. It never occurred to Bohr that particles could simply be guided by waves, as they are in the pilot-wave theory, an idea that, as John Bell says: "seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored." [1, p. 191].

In Heisenberg, one finds the rather "positivistic" view that it is meaningless to speak about things that cannot be observed, an attitude that sometimes leads to a form of Berkeleyan idealism: what is unobservable, hence meaningless, simply does not exist.

But, as Einstein said to Heisenberg in 1926: "it is quite wrong to try founding a theory on observable magnitudes alone. In reality the very opposite happens. It is the theory which decides what we can observe. You must appreciate that observation is a very complicated process. The phenomenon under observation produces certain events in our measuring apparatus. As a result, further processes take place in the apparatus, which eventually and by complicated paths produce sense impressions and help us to fix the effects in our consciousness. Along this whole path [...] we must be able to tell how Nature functions [...] before we can claim to have observed anything at all. Only theory, that is, knowledge of natural laws, enables us to deduce the underlying phenomena from our sense impressions. When we claim that we can observe something new, [...] we nevertheless assume that the existing laws — covering the whole path from the phenomenon to our consciousness — function in such a way that we can rely upon them and hence speak of 'observations'." [6, pp. 63, 65].

Norsen also recounts the various episodes of the Bohr-Einstein debate: the ones at the two Solvay Conferences of 1927 and 1930 and the reply of Bohr to the EPR paper (noting that part of the debate is based on Bohr's recollections in 1949) [2, 3]. In all these exchanges, one can see that Bohr missed the point of Einstein, which, it must be said, only became completely clear in 1935: for Einstein, using locality and states like (1), one can show that ordinary quantum mechanics is an incomplete description of reality. For him, a more complete description, involving "hidden variables" was necessary, irrespectively of whether one can control or predict the values of those variables. But Bohr always took Einstein's objection to mean that one could make predictions that would go beyond the quantum ones. Thus, contrary to a popular misconception, Bohr did not refute Einstein, he simply did not understand what Einstein said.

Actually, the fact that the pilot-wave theory shows that "measurements" are really interactions that do not, in general, reveal any pre-existing property of the particle

(at least for properties other than positions) should be good news for Bohr and his followers, since it vindicates Bohr's intuition about "the *impossibility of any sharp distinction between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear.*" [3, p. 210], quoted in [1, p. 2] (italics in the original). However, that vindication comes as a consequence of the pilot-wave theory and not from some a priori philosophy.

Contemporary physicists are, for better or worse, far less philosophically sophisticated than Bohr or Heisenberg, but also than Schrödinger and Einstein. Although they pay lip-service to the Copenhagen interpretation in their lectures, it is not at all clear what they mean by that, including probably in their own minds.

The physicist David Mermin summarizes what he takes the Copenhagen interpretation to mean today by the slogan: 'Shut up and calculate!' [7]. But this attitude, although prevalent among physicists, is probably doing more harm to science, whose main goal is *to understand* the world, than what all the anti-science folks put together can do.

The Copenhagen interpretation leads us to think that it is simply impossible (because of the intrinsic limitations of our language or of our modes of thinking) or misguided (because it goes against the "spirit of science" understood in a positivistic fashion) to try to get a description of the microscopic world more detailed than the one provided by the quantum formalism.

To this Norsen replies: "But one thing is for sure: to whatever extent the Copenhagen philosophy insists that it is not merely wrong, but impossible, to provide a uniform, coherent, realistic description of the world, which is nevertheless consistent with all known experimental facts, the Copenhagen philosophy is in that regard simply wrong." (p.171). And it is wrong because such a description of the world exists: the pilot-wave theory (and maybe also the spontaneous collapse theory and many-worlds one).

The point of this book is to show that there are indeed ways to escape from the Copenhagen box and, because Norsen explains it so clearly and so patiently, reading this book should be a priority for every physicist who wants to know what the most fundamental physical theory is all about.

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