Original Paper

The concepts of hidden and the concept of partial observation in the quantum mechanics interpretations

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Abstract: The great agreements of the experimental results with the statistical techniques predictions and the Dirac theory influence many interpretations of quantum mechanics. Present work argues the concept of theories of hidden, like the hidden variables, hidden medium, and the hidden geometry. The quantum interpretations produce new quantum mechanics theories instead of explaining the conventional quantum mechanics theory. These theories did not explain the nature of the wave function, then they cannot explain the conventional quantum mechanics. This case does not form a closed cycle of scientific thinking. We proposed an alternative approach via an analogy. It tries to explain the complex nature of a complex vector function, then a form of relativistic quantum mechanics, etc. This project shows that the complex vector function may represent the maximal knowledge.

Keywords: Born's rule; Hidden variables; Hidden thermodynamics, Hidden geometry, Quantum underpinning; Partial observation;

1. Introduction

The complex discerption introduced to the physics with Schrödinger formulation for quantum mechanics [1]. In his work, Schrödinger showed that the wave function is a product of an amplitude factor and a complex phase factor [2], and regarded that the wave function represents a maximal possible knowledge. In his words, Schrödinger said, "Maximal knowledge of a total system does not necessarily include total knowledge of all its parts, not even when these are fully separated from each other and at the moment are not influencing each other at all" [3]. Accordingly, the observable quantum world is the maximal knowledge.

However, the wave function or the quantum state is the first axiom in the quantum postulates list. It is clear that "the axioms of quantum mechanics are silent about the nature of the wave function" [4]. Owing to this silence, physicists who are interested in the foundation of quantum mechanics raise two questions related to the nature of the wave function [5, 6,…]:

- Is the wave function ontic (directly representing a state of reality)? If it is ontic, then exactly what physical state does it represent?
- Is the wave function epistemic (merely representing a state of (incomplete) knowledge)?

The statistical approach of Born rule has a great influence on the both of ontic and epistemic interpretations. The concept of hidden variables stands behind the statistical interpretation. It is the first and more influential concept of statistical foundation interpretations. There are many ontic interpretations of quantum mechanics and are influenced by the hidden variable theories, like de Broglie-Bohm theory [7], Time-symmetric theories [8, 9], and for epistemic like Stochastic interpretation [10].

However, the concept of hidden is not just related to the statistical approach, but the Dirac's relativistic consideration of quantum mechanics has led to another type of hidden. It is the concept of hidden geometry that may stand behind the spin [11].

1.1 *the Scientific thinking*

In the philosophy of science, many researchers look to the function of science as a producer of new concepts, theories [12], and modification of existing theories.
The way that leads to a scientific theory is the scientific thinking. There are many definitions for the scientific thinking. According to Dunbar and Klahr, the "scientific thinking refers to the thought processes that are used in science, including the cognitive processes involved in theory generation, experiment design, hypothesis testing, data interpretation, and scientific discovery" [13].

Those processes of the scientific thinking are started from real phenomena (observation), proposed theory, preparing to test the theory by experiment, and finally the result of investigation return to the original starting step. Thus, the processes can be organized in a cyclic form, and the proved generated theory can be regarded as a scientific discovery. Not Figure1.

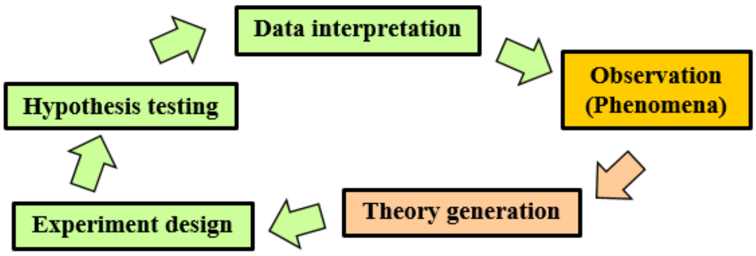

Figure 1 The closed cycle of scientific thinking.

This cycle fits well the classical physics theories. Does it fit the quantum theories as well?

1.2 *the aim*

The article demonstrates the concept of the hidden in quantum mechanics interpretations. We will argue the inability of the hidden variable approach and the hidden geometry to interpret the quantum mechanics. An alternative approach is proposed.

2. The statistical approaches

2.1 Statistical quantum mechanics

The physical utilization of the wave function comes via the Born' rule [14] application. This rule is based on statistical foundation. Owing to the complex nature of the wave function, there is no possibility to a direct physical explanation simulate the real world, and quantum mechanics uses the wave function as a tool without direct explanation. The Born rule provides a link between this mathematical (abstract) formalism and physical meaning. This rule overcomes the problem of mathematical complex representation of the wave function by using complex conjugate multiplication. According to Born "*The reason for taking the square of the* *modulus is that the wave function itself (because of the imaginary coefficient of the time derivative in the differential equation) is a complex quantity, while quantities susceptible of physical interpretation must, of course, be real*" [15].

This statistical picture of Born is supported by Heisenberg's uncertainty principle [16]. According to Born's rule, the probability (P) of finding a particle in space interval ($x = a$, $x = b$) is:

$$P = \int_a^b \psi \, \psi^* \, dx. \tag{1}$$

Where ψ, ψ^* are the wave function of the particle and its conjugate respectively. The wave function is regarded as a probability amplitude, and the probability density function (PDF) is

$$\psi\psi^* \equiv f_\psi(x). \tag{2}$$

It is clear that PDF($f_\psi(x)$) is a real quantity.

However, the same approach is applied in the expectation value technique, as an example the expectation value of the position operator ($\hat{x} = x$):

$$\langle x \rangle = \int_a^b \psi x \, \psi^* \, dx. \tag{3}$$

Predictions of these techniques have high excellent agreements with the experimental results.

However, this statistical approach (prediction of probability) is in excellent agreement with experimental evidence. The Copenhagen interpretation of the quantum mechanics does not exaggerate this rule and regards these probabilities are due to the measurement process. This interpretation is one of many unaccepted interpretations over many years by many physicists. For those physicists, the Copenhagen interpretation "not only does not allow us to think of quantum systems as we do of classical ones, but it just forbids such a thing" [17].

As shown in Figure2 the quantum mechanics does not show a closed cycle of scientific thinking. It looks like a converting tool, from real Hamiltonian and geometry of a system to probabilistic information for the system. It is quite different from the classical cycle of the kinetic theory of gases.

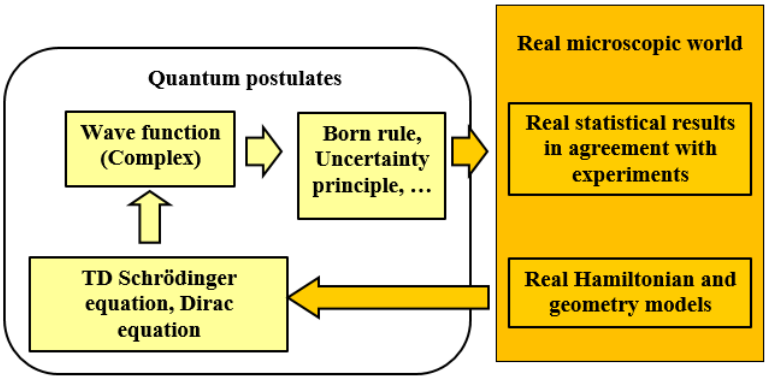

Figure 2 The base of quantum mechanics. There is no cycle of scientific thinking. The input real data and the output is a real probabilistic data.

2. 2 The kinetic theory of gases.

Historically, the kinetic theory of gases is developed according to the observable macroscopic phenomena (fluid) [18]. It started as an assumption when in 1738, Bernoulli derived his law by applying Newton's equations of motion to assumed molecules comprising the gas [19]. The real world, which is here the fluid phenomena led to a theory of large ensemble of molecules, and the theory explains the phenomena via the statistical approach as shown in Figure 3. This cycle starts from the real world and returns to the real world. It is the closed cycle of scientific thinking (or method). This cyclic process is closed, where each step is necessary & sufficient and leads to the next step.

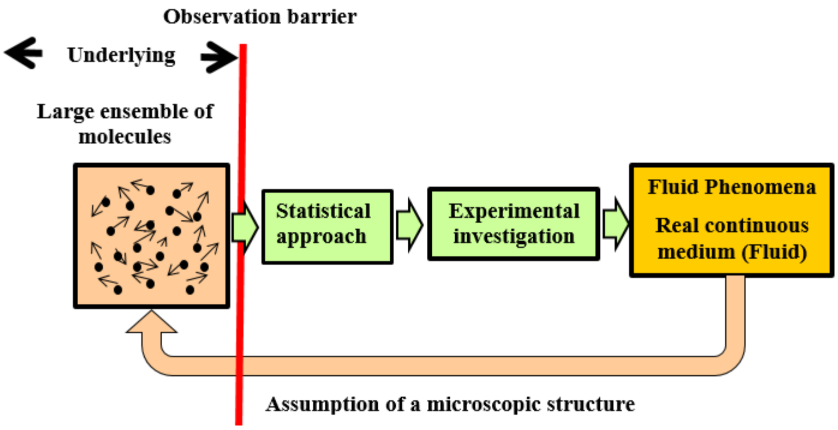

Figure 3 The statistical thermodynamics picture (kinetic theory of gases). The closed cycle of scientific thinking.

As an example for the statistical approach for the kinetic theory of gases is the average quantities. A macroscopic variable like force (F) on a side of the fluid container is:

$$F = \frac{Nm\,\langle v^2 \rangle}{L}. \tag{4}$$

where N, m, v are the number of particles, mass of the particle, the velocity of particle respectively. The hidden variables are of statistical nature [20] and stand behind the macroscopic quantities.

The probability density is another statistical quantity that may relate to the hidden variables. The probability density (P) is an integration of the PDF ($f(x)$) over an interval ($x = a, x = b$):

$$P = \int_a^b f(x)\ dx. \tag{5}$$

The PDF is a fundamental concept in probability theory. The distribution nature of the random variable (hidden variable) is embedded in the PDF, thus the average, probability density and other statistical calculations reflect the whole observable (average) picture for the microstructure. As an example, the PDF of the velocity (v) of molecules in a gas could be the Gaussian distribution:

$$f(v) = \frac{1}{\sqrt{2\pi}} \exp -\frac{v^2}{2}. \tag{6}$$

However, the cycle of scientific thinking is the logical technique that leads to the discovery of the world that lay beyond the fluid phenomena.

2.3 A comparison

It is shown above two approaches of using the statistical technique, the quantum mechanics and the classical kinetic theory of gases. In regarding the statistical approach and the microscopic structure, we have to stand on two serious points.

Firstly, the necessity of classical statistical application is related to its ability to explain the macroscopic phenomena according to an underlying microscopic structure of the macroscopic medium, whereas the necessity of the statistical application in quantum mechanics is related to obtain real physical information [21]. *In other words, the necessity of the quantum statistical approach has not any relation to any underlying microscopic structure.*

Secondly, in classical statistics, the PDF covers all the distribution features. Thus, this technique shows a highest possible picture for the microscopic structure and its relation with the macroscopic level.

In the quantum mechanics, the wave function is a solution of Schrödinger equation. This function is a complex function. Using the function with a statistical technique (Born's rule) leads to regarding it as a probability amplitude (as a square root of the probability density function). Then its modulus squared is equivalent to the probability density function, note Figure 4.

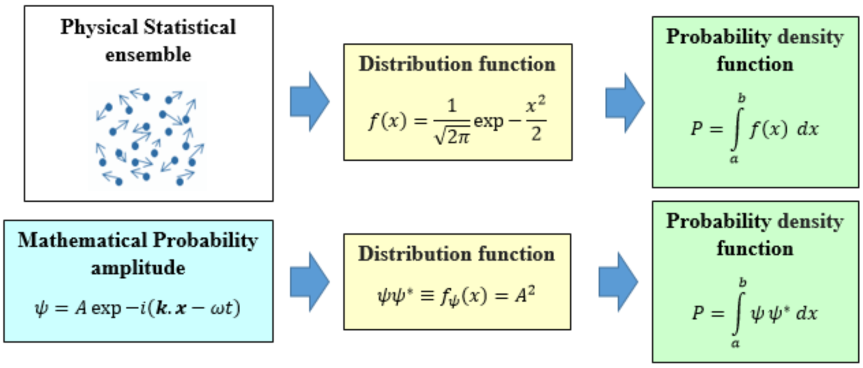

Figure 4 Comparison of the probability in classical (natural distribution as an example) and quantum statistics.

In this point of connection between the probability amplitude and the PDF, we have to note that the PDF does not contain all the information available in the probability amplitude due to the modulus square technique. The statistical provided results are related to the squared amplitude of the wave function only. The complex function (ψ) is composed of a real part and a complex phase factor (Euler’s form) [4]. Born's technique tries to get rid of the complex nature via multiplication with conjugate (ψ^*) [16]. In another hand, the wave function is described as maximal possible knowledge, but the technique of multiplication with conjugate lead to get rid of a part of the maximal possible knowledge.

3. The theories of hidden

The quantum statistical technique is quite efficient in obtaining real information that is related to the real part of the wave function. But, this technique can not reflect any idea about what is behind the nature of the wave function or the probability amplitude. The technique can provide information related to the probability density function.

However, as in classical model (Kinetic theory of gas), the statistical model is related to a fluid model and depends on hidden variables. Many physicists of quantum mechanics adopted such ideas.

3.1 The hidden variables

The quantum statistical approach is based on omission the imaginary part of the probability amplitude due to squaring [15]. In other words, the statistical consideration of the quantum foundation is based on avoiding the complexity nature.

Then we can say that the interpretation theories those based on the statistical approach for the quantum foundation cannot reach the same credibility as that of Born's rule. The statistical technique is quite efficient and in agreement with lab observable phenomena, but, it is not for assumptions for an underlying of the quantum mechanics. If there is an underlying, it must be related to the whole discerption of the wave function.

Accordingly, the classical concept of the hidden variable, which stands behind the PDF can not by playing the same rule in quantum mechanics [22, 23, 24, 25]. In this, de Muynck said

"the idea of a hidden-variables underpinning of quantum mechanics has widely been rejected due to its metaphysical character, hidden variables being thought not to have any observational consequences"[26].

The interpretation theories of statistical nature avoided investigating the complex nature of the wave function. These theories do not explain the wave function, which is used in the statistical approach. Probably, owing to its axiomatic nature, and the axiomatic nature protects this complex enigma.

3.2 The hidden medium

In 1927, de Broglie proposed the concept of pilot wave [27]. In 1956 Bohm and Vigier [28], introduced the hypothesis of the existence of a "sub-quantum medium," which is a hidden medium. Then in 1961, de Broglie developed the "theory of the double solution" [29]. In this theory, the wave equation has two solutions. According to this theory, a particle could be described as a concentrated packet of energy and be guided by a continuous pilot wave (ψ_B) (the external wave). The pilot wave is associated with the particle and described by equation (10) [30]

$$\psi_B = A \exp i(\omega t - kx), \tag{7}$$

where A, ω, and k are a well-determined amplitude, the frequency, and the wave number. The second wave is the internal wave, where the particle undergoes a periodic process similar to a clock with a frequency (in the rest frame)

$$\omega_\circ = \frac{m_\circ c^2}{\hbar}, \tag{8}$$

where $m_\circ$ and $\hbar$ are the rest mass and reduced Plank constant, respectively. The wave function (υ) of the internal wave is [31]:

$$\upsilon = a \exp i\left(\frac{\phi}{\hbar}\right), \tag{9}$$

where a and ϕ are real. To explain this internal periodic process, de Broglie proposed *hidden thermodynamics*.

In this hidden thermodynamics, de Broglie used the classical concept of mass change via the concepts of particle mass and energy [29]. In this work, de Broglie introduced many concepts of hidden things like hidden energy, hidden medium (or sub-quantum medium), hidden thermostat in addition to the hidden thermodynamics. This interpretation takes into account the phase factor.

3.3 The hidden geometry

There is another hidden interpretation of quantum mechanics, but it is less popular than the hidden variables and the hidden thermodynamics. This type does not try to support on the crutch of statistical probability.

Dirac derived his equation through the projective geometry and did not through the algebraic technique. Dirac's biographer Farmelo said “Dirac often said that when he was developing quantum mechanics he used his favourite branch of mathematics — projective geometry — which concerns the relationships between points and straight lines. But why then did he not mention geometry in his early papers?”[32]

Dirac’s matrices imply a mysterious rotation. According to Dirac, these coefficients “describe some new degree of freedom, belonging to some internal motion in the electron” [33], but these coefficients “would not be form invariant with respect to simple spatial rotation” [34].

The trial solution of the Dirac equation is constructed of a combination of a spinor ($u_D(x,t)$) and a complex phase factor. The two factors (spinor and complex phase) are of different conceptual roots, but they form a mathematical structure for the solution (ψ_D). For a free particle, the trial solution has the form

$$\psi_D(x,t) = u_D(x,t)\exp i(k\cdot x - \omega t)\ , \tag{10}$$

where ω , k, and $u_D(x,t)$ are angular frequency, wave vector, and a Dirac four-component spinor respectively. The spinor is neither a scalar nor a four-vector and differs from a tensor. The structure of the spinor is owed to the nature of the Dirac Hamiltonian for the studied case. The complex phase factor is

$$complexphasefactor \equiv \exp i(k\cdot x - \omega t). \tag{11}$$

Dirac described this phase “…this phase is all important because it is the source of all interference phenomena, but its physical significance is *obscure*. So the real genius of Heisenberg and Schrodinger, you might say, was to discover the existence of probability amplitudes containing this phase quantity which is very well *hidden* in nature and it is because it was so well hidden that people hadn’t thought of quantum mechanics much earlier” [35,36]. In Dirac’s words, the hidden is a discerption related to the nature of complex phase factor (and not for the statistical hidden variable).

The terminology of “hidden geometry” came under the Hestenes’s geometric algebra in 1986, as he said “The Dirac theory has a hidden geometric structure. This talk traces the conceptual steps taken to uncover that structure and points out significant implications for the interpretation of quantum mechanics. The unit imaginary in the Dirac equation is shown to represent the generator of rotations in a spacelike plane related to the spin” [11]. Accordingly, Dirac wave function (Eq. 7) reveals a geometric structure and is hidden in the conventional formulation [37]. The geometry is revealed in Dirac theory when it expressed in terms of Spacetime Algebra formalism [11], otherwise, it is hidden.

Hestenes have considered the complex phase factor with a concept of a kinematical model. The kinematical model is a newly proposed concept in the foundation of the wave function. In his approach, Hestenes proposed many concepts related to his kinematical model, like [38, 39, 40]:

- The imaginary *i* can be interpreted as a representation of the electron spin.
- Dirac theory describes a kinematics of electron motion. The kinematical rotation is not necessary to be related to the pair of positive and negative energy states.
- The complex phase factor literally represents a *physical* rotation, the *zitterbewegung* rotation.
- The complex phase factor is the main feature which the Dirac wave function shares with its nonrelativistic limit. Schrödinger wave function inherits the relativistic nature.
- The serious concept in Hestenes proposal is the kinematic origin of the complex phase factor and the physical rotation (Zitterbewegung).

Due to the experimental evidence for the Dirac's theory, Hestenes's theory tries to convert the Dirac's hidden geometry to a revelled geometry. Here as well, Hestenes's theory is influenced by the experimental evidence then it tries to reformulate Dirac theory. Thus, his interpretation produces a new theory. Hestenes's theory does not explain the complex nature of the wave function, as in the case of hidden variable theories.

The hidden geometry interpretation gives a serious role for the complex phase factor and more than that in hidden variables interpretations. So far, this theory has no experimental evidence as in the case of hidden variables, but it did not attract the researchers as in the case of hidden variables interpretations.

4. Is there an alternative?

The theories of hidden do not explain the wave function and tries to propose a new quantum mechanics as in Figure 5. The circle shown in Figure5 of the quantum interpretation does not form a complete cycle of scientific thinking. It tries to avoid the conventional quantum, which is quite powerful and efficient theory.

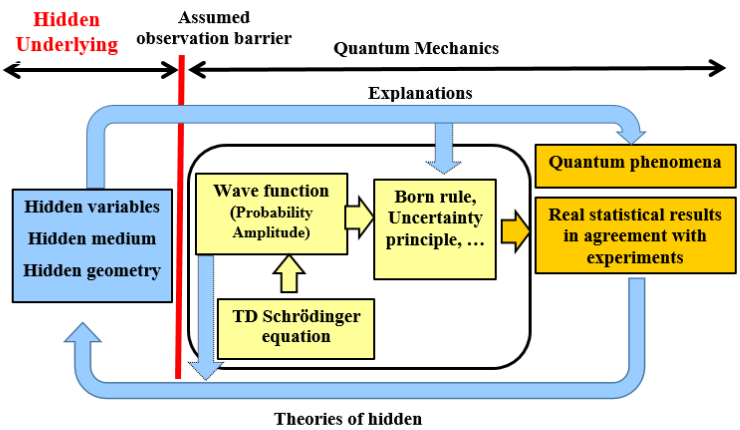

Figure 5 A model for the theories of the hidden cycle.

All the quantum interpretations deal with configuration space, and some propose different forms of complex wave functions (reproduction). As an example, note table 1.

Table 1. Some of the proposed wave functions

	Proposed wave functions
de Broglie [30] The pilot wave	$\psi_B = A \exp i(\omega t - kx),$
de Broglie [31] The wave function (υ) of the internal wave	$\upsilon = a \exp i \left(\frac{\phi}{\hbar}\right)$
Bohm [7]	,
Hestenes [38]	$\psi_H = (\rho \exp i\beta)^{\frac{1}{2}} R$
't Hooft [41] Evolution operator obey this function	$U_{tH}(t_\circ) = \exp -it_\circ H$

These complex forms may be attributed to the axiomatic nature of the quantum state [6]. However, this type of axioms is different from the mathematical axioms. The nature of underlying in these theories is similar to the nature of the abstract microscopic of quantum mechanics. This, type of thinking may be related to the real nature of the underlying of the fluid which is real as well, note Figure 6.

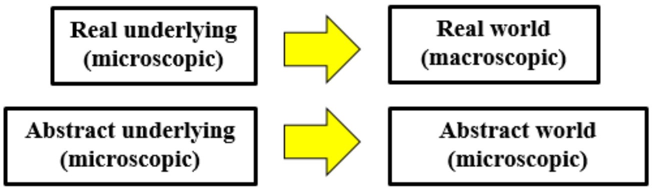

Figure 6 The world and its underlying is of same mathematical nature. The kinetic gas theory and quantum mechanics interpretation theories.

Now can we ask about an interpretation that makes a complete circle of scientific thinking, or that can explain the wave function and accept the conventional theory?
However, our philosophy is:

1. The successful mathematics discerption of nature shows its real (true) observed behaviour.
2. Owing to the great success of the quantum mechanics, the complex nature of the wave function is considered as a real discerption of the observed phenomenon.
3. Since the wave function is real (true) description, then it can be explained mathematically.
4. Explanation of the wave function should lead to explain the quantum phenomena.

Consequently, the conventional quantum mechanics will be part of a complete cycle of scientific thinking, note Figure7. This approach should interpret phenomena like relativistic quantum phenomena, quantum entanglement, wave-particle duality, etc., and unify all the quantum phenomena in a single theoretical model.

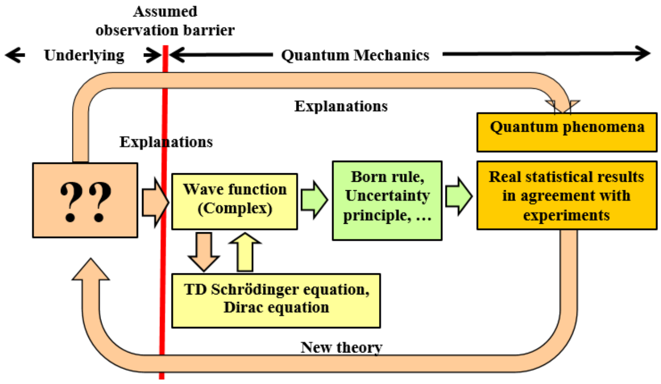

Figure 7 A proposed cycle of an alternative interpretation.

5. An interpretation for the complex phase factor

In 1927, de Broglie proposed the "theory of the double solution" [27], which has two solutions to the wave equation. According to this theory, a particle could be described as a concentrated packet of energy and be guided by a continuous pilot wave (ψ) (the external wave). To explain this internal periodic process, de Broglie proposed hidden thermodynamics in 1961 [29].

In spite of the criticisms of the mainstream physicists, de Broglie's work has attracted and still attracts the attention of a large number of researchers. One of the developments of the de Broglie's work is the Three Wave Model (TWM), which proposed by Kostro in 1978 [42] and is based on two frameworks [43]:

1-The Paris school interpretation of quantum mechanics (de Broglie's works), and

2- Einstein's theory of relativity, where the existence of covariant ether is assumed.

In the TWM, three waves are associated with a micro-object [42]: (i) an internal nondispersive standing wave, (ii) a superluminal wave (the de Broglie wave), and (iii) a subluminal wave.
During the 1980s, the TWM has developed and presented as the Three Wave Hypothesis (TWH) by Horodecki [44, 45]. The TWH implies that a particle having mass is an intrinsically spatially and temporally extended nonlinear wave phenomenon [45], and considers three waves: the Compton, de Broglie, and dual waves. It is clear that it has a fluid base.

Thus, in 2007, Sanduk reconsidered this theory. The TWH is for a massive single particle and not for fluid thus, Sanduk convert the wave formulation to the angular motion formulation. In TWH there are two dispersion relations for the waves. The second change is related to the concept of the single particle. Thus, the two desperation relations were combined in a single formula.
The obtained single formula is quite similar to a classical kinematical model of two perpendicular rolling circles [46, 47]. Here as well there is no experimental prove neither for the TWH nor for the kinematical model.

Even that the kinematical model of the two rolling circles that is related to a relativistic quantum mechanics formulation (TWH), it is a classical model and is without experimental evidence, but it was inspiring. Here, one may ask: if this classical model is related to a relativistic quantum mechanics formulation, so what is its relation with the complex form of quantum mechanics?

In 2012, a system of two rolling circles in a plane was considered with a concept of partial observation to derive a complex vector function [48]. That work was in the classical frame and had nothing to do with quantum mechanics techniques.

5.1 The concept of partial observation

In the special relativity, the observer interests in the definite physical quantities. In other words observable or measured quantities. To achieve this goal, these quantities must be larger than the unit of the used scale, or larger than the used light parameters (λ, f):

$$x \gg \lambda \text{ and } \frac{1}{t} \gg f. \tag{12}$$

The observer in the theory of relativity does not take this condition into account.

Let us apply this condition on a moving point in a circular form with an angular frequency, and the observer uses a monochromatic light of λ, f. $\lambda f = c$. Let us as well consider the following two cases:

First, let the radius of the circle is a_1 and its angular frequency is ω_1. The linear velocity of the point is $v_1 = a_1\omega_1 \ll c$. The observer cannot recognise the circle when $a_1 \ll \lambda$, but the frequency is recognised $(\omega_1 \gg \omega = 2\pi f)$. The observer faces a problem. Ha/her can recognise a point with an angular frequency (spin), but of zero radius $(a_1 = 0)$, note Figure 6.

Second, let the radius of the circle is a_2 and its angular frequency is ω_2. The observer cannot recognise the angular frequency when $\omega_2 \ll \omega = 2\pi f$, whereas the radius can be recognised ($a_2 \gg \lambda$). Here as well, there is another problem. This consideration faces the problem of the circular location of a point without angular frequency $(\omega_2 = 0)$. Here the observer can see the point at rest and at different locos of the circle! This consideration faces the problem of the circular orbit with zero angular velocity $(\omega_2 = 0)$, note Figure 8.

This partial observation works like a fishing net. The observables are larger than the net cell.

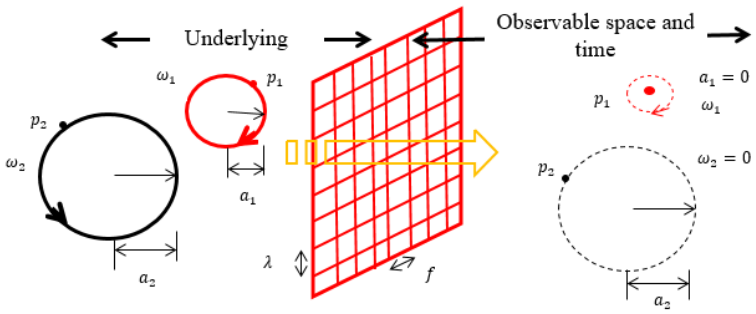

Figure 8 Effects of the partial observation.

In order to make a monochromatic light satisfies the two cases above, the radii of the two circles are:

$$a_1 \ll a_2. \tag{13}$$

The two circles may be arranged as in Figure 9. This is similar to that of Ref.48. The work of 2012 [48] proposed a system of two rolling circles, of two different radii.

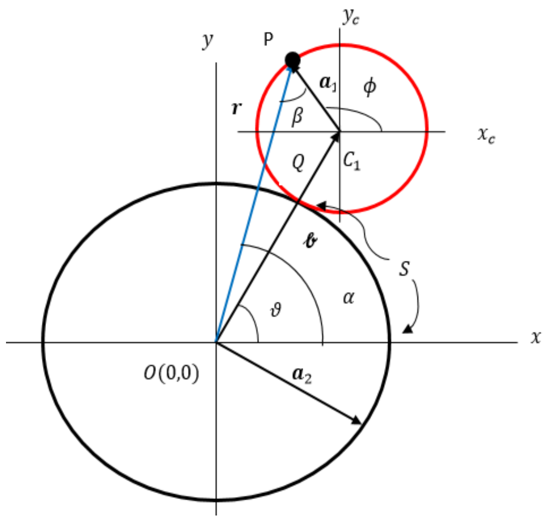

Figure 9 The two rolling circles system.

In this system, the position vector (r) of a point (p) on the first circle is:

$$r = b\left\{\cos(\vartheta - \phi + \beta) \pm \sqrt{-sin^2(\vartheta - \phi + \beta) + (\frac{a_1}{b})^2}\right\}. \quad (14)$$

The ratio of the system is:

$$\frac{a_2}{a_1} = \frac{\phi}{\vartheta} = \frac{\omega_1}{\omega_2} = \mu > 1. \quad (15)$$

Eq. (15) looks like a gear ratio.

Let us consider that, there are two lab observers and both know that the position vector of the mathematical model (Figure7) is defined by the Eq. (14). In case of that, this model is a real material model, and in order to observe it, the lab observers need to use a light. Let both of them use monochromatic lights of two different wavelengths. The lab observer who uses light of $\lambda \ll a_1 \ll a_2$ and $\omega \ll \omega_2 \ll \omega_1$ will recognise the model and its motion completely, and apply Eq. (14) to find the position vector. The other observer who uses light of $a_1 \ll \lambda \ll a_2$ and $\omega_2 \ll \omega \ll \omega_1$ will not recognise the model completely (as in the cases above), and his application for Eq. (14) shows that:

$$Z = b\left\{\cos(\vartheta - \phi) \pm \sqrt{-sin^2(\vartheta - \phi)}\right\} = b\ exp \pm i(\vartheta - \phi). \quad (16)$$

This equation shows a transformation from real space to Hilbert space ($r \rightarrow Z$), and that owing to the partial observation of the system of the two rolling circles.
This approach may show a relationship between the kinematical model and the complex phase factor. Figure 10 shows this transformation.

The concept of relativity is related to the relativity of velocity or motion. In regarding the partial observation in a relativistic way, we can say there may be relativistic existence according to the observation condition.

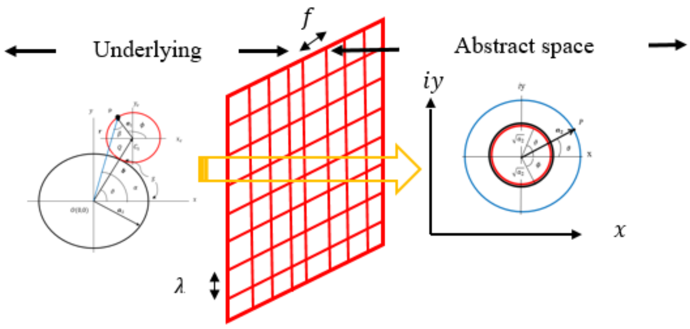

Figure 10 Effects of the partial observation on a two rolling circles system.

6. The kinematic underpinning

For the above-described model, Can this kinematic model of the two rolling circles under the partial observation analogize the relativistic quantum mechanics? Such a concept is out of the quantum mechanics techniques, and according to this approach, the quantum mechanics can be regarded as an emergent.

This approach is the subject of the next article. The analogies will be used to bridge between the relativistic quantum mechanics, and what the above model tries to explain.

References

1. Schrödinger, E.: An Undulatory Theory of the Mechanics of Atoms and Molecules, *Phys. Rev*., 28 (6), pp 1049–1070 (1926).
2. Moore, W.: *Schrodinger, Life and Thought*, Cambridge University Press, 1993, p197-220.
3. Schrödinger, E. (1935) "*The Present Situation in Quantum Mechanics*" trans. by J.D. Trimmer, in Wheeler and Zurek, 1983, pp. 160 (1983).
4. Charrakh, C., On the Reality of the Wave function, 2017 *arXiv*:1706.01819v1 [physics.hist-ph] (2017).
5. Gao, S., Meaning of the wave function: In search of the ontology of quantum mechanics, *arXiv*:1611.02738v1 [quant-ph] (2016).
6. Der Pluijm, H., *On the interpretation of the quantum wave function*, Master's thesis, Radboud University Nijmegen, (2016).
7. Bohm, D., A Suggested Interpretation of Quantum Theory in Terms of Hidden Variables. I, *Phys. Rev*., 85, 2, PP.166-193 (1952).
8. Watanabe S., Symmetry of physical laws. Part 1 and 2. Q-number theory of space-time inversions and charge conjugation Rev. Mod. Phys. **27** 40-76 (1955).
9. Watanabe S.,Symmetry of physical laws. Part 3. Prediction and retrodiction Rev. Mod. Phys. **27** 179-86 (1955).
10. Nelson, E., "Derivation of the Schrödinger Equation from Newtonian Mechanics". *Phys. Rev*. **150**: 1079–1085 (1966).
11. Hestenes,D., Clifford Algebra and the Interpretation of Quantum Mechanics, in: Clifford Algebras and Their Applications in Mathematical Physics, Editors J. S. R. Chisholm & A.K. Common, Springer, Dordrecht, NATO ASI Series, vol 183, pp 321-346 (1986).
12. Bruner, J.S., Goodnow, J.J. & Austin, G.A., A study of thinking. New York: NY Science Editions (1956).
13. Dunbar, K., and Klahr, D., Scientific Thinking and Reasoning ,The Oxford Handbook of Thinking and Reasoning Edited by Keith J. Holyoak and Robert G. Morrison, Oxford University Press. Ch 35, (2012).
14. Born, M., Zur Quantenmechanik der Stoßvorgänge, *Zeitschrift für Physik*, 37, No.12, pp. 863–867 (in German) (1926).

15. Born, M., *Atomic Physics*, Blackie & Son Limited, London-Glasgow, seventh edition, 1963; Mir, Moscow, (1965).
16. Heisenberg, W., "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik", *Zeitschrift für Physik* (in German), 43 (3–4): 172–198 (1927).
17. Sanz, A., Quantumness beyond quantum mechanics, EmerQuM 11, Journal of Physics: Conference Series **361** 012016 (2012).
18. Kauzmann, W., *Kinetic Theory of Gases*, Dover Books on Chemistry, Courier Corporation, Ch.2 (2013).
19. Bernoulli, D., Kinetic Theory of Gases, *The World of Mathematics*, J. R. Newman, ed., Volume 2 , Simon and Schuster, New York, p. 774, (1956).
20. Elidan, G., Lotner, N., Friedman, N., and Koller, D., Discovering Hidden Variables: A Structure-Based Approach. In Leen *et al*., pages 479-485 (2001).
21. Born, M., *Atomic Physics*, Blackie & Son Limited, London-Glasgow, seventh edition, 1963; Mir, Moscow, (1965).
22. Ballentine, L. E.,"The statistical interpretation of quantum mechanics," *Reviews of Modern Physics*, vol. 42, no. 4, pp. 358–381, (1970).
23. Vaidman, L., Quantum theory and determinism**,** Quantum Stud.: Math. Found., Volume 1, Issue 1-2, pp 5–38 (2014).
24. Accardi, L.: Topics in quantum probability. Phys. Rep. **77**, 169 (1981).
25. Fröhlich, J., Schubnel, B.: Quantum probability theory and the foundations of quantum mechanics, arXiv:1310.1484 (2013).
26. De Muynck, W.M., Foundations of quantum mechanics, an empiricist approach, Kluwer Academic Publishers, Dordrecht, Boston, London, Ch.10, ISBN 1-4020-0932-1 (2002).
27. De Broglie, L. "La mécanique ondulatoire et la structure atomique de la matière et du rayonnement". *Journal de Physique et le Radium* , 8 (8): 225–241 (1927).
28. Bohm, D., Vigier, J.P.: Model of the causal interpretation of quantum theory in terms of a fluid with irregular fluctuations. Phys. Rev. 96, 208{216 (1954).
29. De Broglie, L., *The Thermodynamics of the isolated particle*, The Great Problems of Science, No.17, Translated by D. H. Delphenich, Gauthier-Villars Editor, Paris, (1964).
30. De Broglie, L., Une tentative d'interprétation causale et non linéaire de la mécanique ondulatoire: la théorie de la double solution, Gauthier Villars, Paris, (1956).
31. De Broglie, L., Interpretation of quantum mechanics by the double solution theory, *Annales de la Fondation Louis de Broglie*, 12, 4, pp 1-23 (1987).
32. Graham Farmelo, Dirac's hidden geometry, Nature, Vol 437, No. 7057, P. 323, 15 September (2005).
33. Dirac, P. A. M.: Principles of Quantum Mechanics, 4th edition, Oxford University Press, Ch. 11 (1958).
34. Greiner, W.: Relativistic Quantum Mechanics: Wave Equation, 3rd edition, New York: Springer-Verlag Berlin Heidelberg, **107** (2000).
35. Yang, N. C., *Schrödinger: Centenary Celebration of a Polymath*, Ed. C. W. Kilmister, Cambridge University Press, P. 53 (1989).
36. Dirac, P.A.M., Relativity and Quantum Mechanics, *Fields & Quanta*, 1972, 3, 139- 164.
37. Hestenes, D., and Weingartshofer, A., *The Electron, New Theory and Experiment,* Kluwer, Dordrecht (1991).
38. Hestenes, D., On Decoupling Probability from Kinematics in Quantum Mechanics, P. F. Fougere (ed.), Maximum Entropy and Bayesian Methods, Kluwer Academic Publishers, 161-183 (1990).
39. Hestenes, D., Space-Time Algebra, Gordon & Breach, New York (1966).
40. Hestenes, D., The Zitterbewegung Interpretation of Quantum Mechanics, Found. Phys. 1990, 20, 1213–1232 (1990).

41. 't Hooft, G.,Quantum Mechanics from Classical Logic,EmerQuM 11: Emergent Quantum Mechanics 2011, Journal of Physics: Conference Series **361** 012024 (2012).
42. Kostro, L., A wave model of the elementary particle, a three waves hypothesis, unpublished paper sent to L. de Broglie in the French version (1978).
43. Kostro, L., A three-wave model of the elementary particle, Physics letters, **107A**, number 9,7, pp. 429-434 (1985).
44. Horodecki, R., De Broglie wave and its dual wave, Phys. Lett. 87A, pp. 95-97 (1981).
45. Horodecki, R., Superluminal singular dual wave, Lett. Novo Cimento 38, 509-511 (1983).
46. Sanduk, M. I., Does the Three Wave Hypothesis Imply a Hidden Structure?, Apeiron, 14, No.2 pp. 113-125 (2007).
47. Sanduk, M. I., Three Wave Hypothesis, Gear Model, and the Rest Mass, *arXiv*:0904.0790 [physics.gen-ph], (2009).
48. Sanduk, M., A kinematic model for a partially resolved dynamical system in a Euclidean plane, *Journal of Mathematical Modelling and Application*, 1, No.6 pp. 40-51. ISSN: 2178-2423 (2012).